Review in Sound Absorbing Materials

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ABSTRACT

This article is a bibliographical revision concerning acoustic absorbing materials, also known as poroelastics. These absorbing materials are a passive medium used extensively in the industry to reduce noise. This review presents the fundamental parameters that define each of the parts comprising these materials, as well as current experimental methods used to measure said parameters. Further along, we will analyze the principle models of characterization in order to study the behaviour of poroelastic materials. Given the lack of accuracy of the standing wave method, three absorbing materials are characterized using said principle models. A comparison between measurements with the standing wave method and the predicted surface impedance with the models is shown.

KEY WORDS: poroelastic material, acoustic surface impedance, poroelastic characterization, models, sound absorption

1. INTRODUCTION

The number of products that include a low noise level design increases daily. However, apart from the design itself, it is frequently necessary to use techniques that lower the level of noise in the product or industrial application.
A variety of methods are available for noise reduction but they can be basically grouped as follows: passive and active mediums. Active mediums differ from passive mediums in that it is necessary to apply external energy in the noise reducing process.

The absorbing materials, as such, are passive mediums that lower noise by disseminating energy and turning it into heat. Acoustic absorption depends on the frequency of the sound waves. In porous materials at high frequencies, an adiabatic process takes place that produces heat loss due to friction when the sound wave crosses the irregular pores. On the other hand, at low frequencies, poroelastic materials absorb sound by energy loss caused by heat exchange. This is an isothermal process. In general, poroelastic efficiency is limited to high frequencies.

The absorption phenomenon differs from that of insulation or shock absorption. This process causes a vibrating movement to diminish in size with time. The origin can vary: due to friction between two surfaces, as a result of internal friction or hysteresis of the material itself, etc.

Other passive mediums exist, such as resonators that reduce noise by transforming it in the vibration of the resonator itself. A noise resonator is nothing more than a system that begins to vibrate due to variations in sound pressure; the resonator begins to vibrate and produces losses in the form of heat. Resonators can be modelled as a forced and damped system with one degree of freedom, with an equivalent excitation strength, equivalent mass element, equivalent spring element and equivalent damping element.

The equivalent strength is produced by the variations in sound pressure and the mass, spring and damping elements depend on the resonator to be modelled: the type of resonator, its dimensions, materials, etc. The mass consists of everything that is subject to movement and the spring to all that provides stiffness. The damping consists of everything that causes energy loss in the moving system.

There are different types of resonators: Helmholtz resonators, panel resonators and punched plate resonators. Currently, active type resonators exist that vary their geometry depending on the variation of external noise to be lowered.

There are also solutions consisting in actuators integrated in absorbing plates that are capable of producing vibrations that counter the original vibration (Nykänen, 2001).

On the other hand, in active noise control, it is necessary to have a noise or vibration meter based on various sensors in order to act on the system via adequate actuators to cancel the noise or
vibrations. This is why it is also necessary to have a form control to close the reader-control-actuator loop. Therefore, the various control algorithms will play a major role in active noise control.

It is known that passive mediums act correctly at high frequencies while noise reduction in low frequency ranges require the introduction of active noise control techniques. The efficacy of each medium at different frequencies is shown in Figure 1.

![Figure 1: Range of working frequencies for active and passive mediums.](image)

(Landaluze, Portilla, Pagalday, Martínez and Reyero, 2003).

This bibliographical review focuses on the study of porous materials, and other passive mediums such as resonators or active mediums that combine actuators with absorbing materials are out of the scope of the present review. A standard technique to characterize these absorbing materials is the impedance tube. There have been a number of works into the accuracy of the standing wave tube method. Lauriks et al (1990) observed that only a free-field technique for the normal surface impedance could give valid results and proved that the standing wave method (or classical Kundt tube method) only roughly corresponded to the results obtained in the free field because the friction between the screen of the absorbing material and the tube caused extra attenuation. Allard et al (1991) also used a free-field technique to measure the normal surface impedance because the Kundt tube is not very accurate for resonant materials. Vigran et al (1997) carried out a comparison between measurement results of the acoustic absorption coefficient and impedance using two different methods, the standing wave tube and a free field method. They analyzed the effect of the constraints at the tube wall on the absorption coefficient of an elastic porous material. They conclude that the
friction between the material and the tube was to stiffen the material and proved that impedance tube measurements should be used carefully in cases where the elasticity of the frame contributed to the surface impedance of the material. Pilon et al (2003) defined a new parameter named as Frame Acoustical Excitability and established a critical value below which the theoretical absorption could be efficiently measured using a standing wave tube. Recently Horoshenkov et al (2007) have carried out reproducibility experiments on the inter-laboratory characterization of the acoustical properties of three types of consolidated porous media. As main conclusions, they state that:

“The existing ISO10534-21 should be revised to define more precisely: (i) the procedure for sample preparation and minimum number of tested specimen; (ii) the minimum size of the sample as a function of the material density, bulk modulus of the material skeleton and flow resistivity; (iii) the sample mounting conditions; (iv) the type of stimuli and signal processing method; and (v) the procedure for merging material data obtained in tubes of different diameters, a procedure that has not been discussed here. The revised standard procedure should enable quantification of the intrinsic experimental errors.”

Therefore, since the agreement between the free-field measurements and the theoretical calculations based on Biot theory are good (Lauriks et al, 1990), this article will focus on describing two aspects in detail: the primary parameters that define absorbing materials and the characterization of said materials. The characterization of poroelastic materials includes the description of the parameters necessary to define the behaviour of the fluid in the pores, the skeleton of solid phase of the material and the coupling mechanism between both phases. In addition to the description of the parameters, we explain the tests necessary for their quantification. In like manner, we include different empiric and phenomenological models of mechanical behaviour of porous materials. In addition, we will show the agreement between Kundt’s tube measurements and theoretical calculations of the surface impedance for three absorbing materials: Acustec, Acustifiber P and Acusticell. Acustec is a material made of mineral wool with a high mechanical strength, meanwhile Acustifiber P is made of polyester fibre and Acusticell is an absorbing expanded polyurethane foam.
2. DESCRIPTION OF POROELASTIC MATERIALS

Poroelastic materials are divided into two phases: a solid phase known as the skeleton and a liquid phase that is normally air.

There are different types of poroelastic materials: fibrous, such as cloth and rock wool, foam, granular and vegetable binders such as straw.

Porous materials absorb acoustic energy by friction with the air that moves inside the pores. The details of the various structures of porous materials are shown in Figure 2.

In order to characterize porous materials, it is necessary to know the parameters that define each part of poroelastic materials, that is, the liquid, the skeleton and the coupling between both of these phases. These parameters are described below.

2.1 Parameters that define liquid properties

The liquid parameters necessary for correct characterization of absorbing material are found below.
2.1.1 **Volume mass**

Volume mass, \( \rho_0 \), represents the mass of a body per unit of volume and is generally expressed in \( \text{kg/m}^3 \).

2.1.2 **The ratio of specific heats**

The parameter, \( \gamma \), ratio of specific heats, is the quotient of specific heats at a pressure of, \( C_p \), and constant volumes, \( C_v \), \( \gamma = C_p / C_v \).

The specific heat of a body represents the amount of calorific energy that a substance must be supplied with per unit of mass to increase its temperature by one degree. Therefore, the units in the international system are \( \text{J/(kgK)} \) (Franco, 2004).

The ratio between two specific heats are given by: \( C_v = C_p - r \), where \( r \) is the specific constant of perfect gases that depends on the general constant of perfect gases and the molecular weight of the specific gas. For air, \( r = 287 \text{ J/(kgK)} \). In general, specific molar heats are used in order not to include molecular weight.

In the case of air, \( \gamma = 1.4 \), considered as an ideal diatomic gas.

2.1.3 **Sound velocity**

Acoustics assume that the process of propagation is isoentropic. In the case of perfect gases (or their mixture, as in air), isoentropic processes are characterized by the law \( p = C \rho^\gamma \) where \( p \), is pressure, \( \rho \) is the density of the medium, \( \gamma \) is the quotient of the specific heats at constant pressure and volume and \( C \) depends on the gas and also on the value (constant) of the specific entropy during the process (Kinsler et al, 2000).

The speed of sound, \( c \), is the speed at which a pressure wave travels in this medium. It is known that this speed is equal to the square root of the partial derivative of the pressure with respect to the density, when entropy remains constant, \( s_0 \), that is,
\[
c = \sqrt{\frac{\partial \hat{p}}{\partial \rho}} (\rho_0, s_0), \tag{1}
\]

where, \( p = \hat{p}(\rho, \theta) = \hat{p}(\rho, s) \), \( \theta \) denotes absolute temperature and \( s \) specific entropy (that is, entropy per unit of mass).

\[
\frac{\partial \hat{p}}{\partial \rho} (\rho_0, s_0) = \gamma \frac{\hat{p}}{\rho} (\rho_0, s_0). \tag{2}
\]

Substituting the values for air at 0º C and 1 atm of pressure:

\[
c_o = (1.402 \times 1.01325 \times 10^5 / 1.293)^{\frac{1}{2}} = 331.5 \text{ m/s}.
\]

For the particular case of perfect gases, the speed of sound can be estimated from its status equation: \( p = \rho r T_K \) where \( \rho \) is density and \( r \) is the specific constant of perfect gases.

\[
c^2 = \frac{\partial \hat{p}}{\partial \rho} (\rho_0, s_0) = \gamma \frac{\hat{p}}{\rho} (\rho_0, s_0) = \gamma r T_K. \tag{2}
\]

For a temperature of 0 ºC, speed of sound is: \( c_o = (1.402 \times 287 \times 273.15)^{\frac{1}{2}} = 331.5 \text{ m/s} \).

The speed of sound in a medium is also defined by the bulk modulus of the medium as:

\[
c = (K / \rho)^{\frac{1}{2}}, \tag{3}
\]

where the bulk modulus is:

\[
K = \rho \frac{\partial \hat{p}}{\partial \rho}. \tag{4}
\]

We must take into account that the status equation of perfect gases greatly depends on temperature, so that the response to efforts of compression depends on what happens to the temperature during the process.

Two extreme cases can be considered. One in which the liquid compression process is performed very slowly, resulting in an exchange of heat with the skeleton and maintaining constant temperature. The response of the liquid to the compression effort will be determined by the isothermal compressibility modulus:
\[ K_T = \rho \frac{\partial p(\rho, T)}{\partial \rho}. \]  

(5)

However, if the process is very fast, no heat exchange will take place. Therefore, the process will be adiabatic, that is, at constant entropy and the response of the fluid to the compression effort will be determined by the adiabatic compressibility modulus.

\[ K_a = \rho \frac{\partial p(\rho, s)}{\partial \rho}. \]  

(6)

Both moduli, isothermal and adiabatic, are related by the ratio of the specific heats, \( \gamma \), as follows:

\[ K_a = \gamma K_T. \]  

(7)

### 2.1.4 Dynamic viscosity

Viscosity represents the resistance to flow of a fluid. The common unit of dynamic viscosity, \( \eta \), is the centipoise (cp), while the official unit is the Pascal second (Pa.s) equivalent to 1000 centipoises. The dynamic viscosity of the air at 0 ºC is 1.72x10^{-5} Pa.s.

### 2.1.5 Prandtl number

This is the quotient between kinematic viscosity and thermal diffusivity. Kinematic viscosity is the dynamic viscosity divided by density, while thermal diffusivity has to do with the thermal inertia of a body (greater or lesser capacity of a body to homogenize its temperature). If thermal diffusivity is high, thermal inertia is low and therefore, homogenization of body temperature will be rapid.

Prandtl's number, \( P_r \), (Allard, 1993) is an adimensional number defined according to

\[ P_r = \frac{\eta / \rho}{\kappa / \rho C_p} = \frac{\eta C_p}{\kappa}, \]  

(8)

where \( \kappa \) is the thermal conductivity measured in watt/mºK.

\( P_r \) of air at 0 ºC is equal to 0.708.
2.2 Parameters that define the properties of the skeleton of the porous material

According to Langlois et al (2001), and following on Biot’s theory, the skeleton of an open-cell, poroelastic isotropic material can be defined by three elastic properties: The shear modulus \((G)\), Young’s modulus \((E)\) and Poisson’s modulus \((\nu)\). These three properties are generally complex and dependent on frequency due to skeleton viscosity. Simulations and experimental measurements performed on multi-layer materials show that, although in some cases the dynamic modulus \((G \text{ or } E)\) can be greater than its static value, the use of constant elastic properties, measured at low frequencies, are well correlated (Allard, 1993, and Panneton, 1996, taken from Langlois et al, 2001). Therefore, to characterize these properties, it is necessary to perform a dynamic test, but in most acoustic and vibro-acoustic problems it is sufficient to perform quasistatic measurements of the properties mentioned (Mariez et al, 1996, taken from Langlois et al, 2001).

For open pore poroelastic materials, this characterization must be performed in a vacuum, since the movement of the fluid through the pores can affect the measurement (Ingard, 1994, taken from Langlois et al, 2001). There are also numerical methods that attempt to eliminate these fluid effects numerically (Melon et al, 1998, taken from Langlois et al, 2001).

Another aspect to be considered when dealing with porous materials is their anisotropy. According to Dauchez (1999) and Dauchez et al (2000), taken from Langlois et al. (2001), when the degree of anisotropy of the material is not very high, it is shown that isotropic models show better results than anisotropic models.

Numerous methods have been proposed to measure these properties. Some of them are based on uniaxial compression tests or transmissibility tests applied to a disc of material, in the assumption that it is equivalent to a spring with no mass or to a spring and mass system (Kim and Kingsbury, 1979; Wijesinghe and Kingsbury, 1979 and Okuno, 1986, taken from Langlois et al., 2001). Other methods such as shear tests use brick type materials (Hilyard and Cunningham, 1994, taken from Langlois et al. 2001). The measurements in these methods do not represent the true properties of the material due to the fact that the surrounding conditions are not taken into consideration. The validity of this approximation is inversely proportional to the size of the sample. To minimize the effect of the surrounding conditions Pritz (1982), taken from Langlois et al. (2001) proposes a method bases on a long slim sample of material. By doing so, Young's modulus would be determined by longitudinal
vibrations and Poisson's modulus would be determined by the variation in diameter of the sample. In practice, the use of this long, slim sample is the limitation of this method, as it is not easy to obtain this type of geometry in porous commercial materials. On the other hand, obtaining disc or brick type samples is easier.

So, to take into account the effects of sample size and those of the surrounding conditions, Mariez et al (1996) and Sim and Kim (1990), taken from Langlois et al. (2001) have investigated characterization methods based on finite elements.

In the work by Mariez et al. (1996), a cubic sample was compressed between two rigid plates. Afterwards, Young’s modulus \( E(\omega) \) and Poisson’s modulus \( \nu(\omega) \) were adjusted in the finite element model until mechanical impedance \( F(\omega)/d_1(\omega) \) was obtained and the quotient between displacements \( d_2(\omega)/d_1(\omega) \) measured in the experiment at angular frequency \( \omega \). Where \( F(\omega) \) is the force applied to the sample, \( d_1(\omega) \) is the vertical displacement of the sample and \( d_2(\omega) \) is horizontal displacement.

Angular frequency, \( \omega \), will be sufficiently low for the quasi-static model to be a good approximation of the dynamic model. The method is applied for much lower angular frequencies than the first resonance of the sample. Section 2.2.1 presents the details of the experiments undertaken by Mariez et al (1996).

However, the work by Sim and Kim (1990) is based on a transmissibility test using two discs with a different form factor. Form factor is defined as \( R/2L \), where \( R \) is the sample radius and \( L \) the thickness of the sample. The first disc has a small form factor, so a good estimate of Young’s modulus can be obtained from the measurement of transmissibility. Afterwards, a finite element model was constructed to simulate transmissibility of the second sample. Contrary to the first sample, the second sample has a large form factor. In order for the simulated transmissibility to coincide with the measurement, Poisson's modulus is adjusted. This modulus is used to repeat the estimate of Young’s modulus of the first sample and iteration is continued. Iteration is interrupted when \( E \) and \( \nu \) have small changes and practically stable.

The greatest limitation of these methods is that a finite element model has to be performed and solved iteratively with the time increase this supposes.
After this bibliographic review of the experimental methods of characterization of porous material skeleton properties, we will comment the methods used by Mariez and Sahraoui (1997) and Langlois et al. in detail (2001).

2.2.1 *The Mariez and Sahraoui method*

With the methodology by Mariez and Sahraoui (1997) presented below, we can obtain complex Young and Poisson modulus with a quasi-static compression test and with an inverse problem that is solved with finite elements. This methodology can be applied in both isotropic and anisotropic materials.

2.2.1.1 Isotropic materials

In this method (Brouard et al, 2004), the poroelastic material in the shape of a disc is placed between two rigid plates (see Figure 3).

![Diagram of the compression test.](image)

The face of the plates is covered with sandpaper to prevent the radial slipping between the discs. The sample is compressed using an exciter, (see Figure 4), and vertical displacement \(d_1(\omega)\) is measured using a displacement meter while horizontal displacement \(d_2(\omega)\) is measured with a laser vibrometer.
From this experiment we obtain:

- Sample stiffness

\[ K^*(\omega) = \frac{F(\omega)}{d_1(\omega)} \]  

(9)

- The transfer function

\[ T^*(\omega) = \frac{d_2(\omega)}{d_1(\omega)} \]  

(10)

Where,

- \( F(\omega) \) is the force applied on the lower plate.
- \( d_1(\omega) \) is the vertical displacement of the lower plate measured with an inductive transducer.
- \( d_2(\omega) \) is the horizontal displacement measured with a laser vibrometer.

For isotropic porous materials \( K^*(\omega) \) and \( T^*(\omega) \) depend on \( E(\omega) \) and \( \nu(\omega) \):

\[ K^*(\omega) = K[E(\omega), \nu(\omega)] \]  

(11)

\[ T^*(\omega) = T[E(\omega), \nu(\omega)] \]  

(12)

To obtain values of \( E(\omega) \) and \( \nu(\omega) \), a numerical simulation is performed using finite elements.
2.2.1.2 Anisotropic materials

If the porous material is considered anisotropic (Mariez and Sahraoui, 1997), to characterize porous material with an axisimmetrical shape, five complex, independent functions are needed: $E_L(\omega)$, $E_T(\omega)$, $G_{LT}(\omega)$, $\nu_{LT}(\omega)$ and $\nu_{TT}(\omega)$, where longitudinal and cross-section directions are named L and T, respectively.

These five functions are experimentally defined in the frequency domain, thanks to mechanical impedance ($K(\omega)$) and the transfer function ($T(\omega)$) that is obtained from the experimental measurements (described for the isotropic case).

After the experiments, nine elastic constants that are functions of frequency are obtained from the skeleton of the porous material:

<table>
<thead>
<tr>
<th>Load directions (Primary axes)</th>
<th>Modulus</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (Longitudinal)</td>
<td>$K_{LL}$</td>
<td>$T_{LT}$ and $T_{LT'}$</td>
</tr>
<tr>
<td>T (Transversal)</td>
<td>$K_{TT}$</td>
<td>$T_{TL}$ and $T_{TT}$</td>
</tr>
<tr>
<td>T' (Transversal)</td>
<td>$K_{TT'}$</td>
<td>$T_{TL'}$ and $T_{TT'}$</td>
</tr>
</tbody>
</table>

Table 1: Functions obtained from experimental measurements.

Four of these parameters ($K_{TT'}, T_{LT'}, T_{TL}$ and $T_{TT'}$) are used to confirm transversal isotropy. Afterwards, vector $[K_{LL}, K_{TT}, T_{LT}, T_{TL}, T_{TT}]$ can be used to obtain vector $[E_L, E_T, G_{LT}, \nu_{LT}, \nu_{TT}]$, resolving an inverse problem iteratively with finite elements.

2.2.2 Quasi-static Christian Langlois et al. method

The originality of the quasi-static Christian Langlois et al. method (2001) lies in the polynomial relations between the compression modulus [that will be defined in equation (18), Young’s modulus, Poisson’s modulus and the shape factor. The relations are established in a low frequency range, under the first resonance. Contrary to the methods explained until now, this methodology only requires a single measurement, called mechanical impedance. It should also be noted that no iteration is performed with any of the finite element models during the characterization of the properties.
The configuration of Langlois et al. (2001) is the same as that of Mariez and Sahraoui (1997), shown in Figure 3.

From this experiment, we obtain the transfer function or mechanical impedance defined in the following equation (13):

\[ Z_m(\omega) = \frac{F(\omega)}{d_1(\omega)}. \]  

(13)

Where, \( F(\omega) \) is force measured in the upper plate, \( d_1(\omega) \) is the displacement of the lower plate.

To calibrate the system, a calibrated spring is placed between the two rigid plates. Spring stiffness should be such that its resonance frequency is higher than the maximum of the band analyzed. In this range of frequencies, it is assumed that stiffness is constant and equal to its static stiffness value \( k_0 \). Therefore, the correction function \( H_c \) of the experimental measurements is defined as:

\[ H_c(\omega) = \frac{1}{k_0} \left| \frac{F(\omega)}{d_1(\omega)} \right|_{\text{spring}}. \]  

(14)

Therefore, the equation (13) is:

\[ Z_{mc}(\omega) = \frac{Z_m(\omega)}{H_c(\omega)}. \]  

(15)

At low frequencies, where fluid flow and thermal process are insignificant and applied deformation is less than 5% (Hilyard and Cunningham, 1994 taken from Langlois et al. 2001), it is assumed that the measurements in the air are similar to the measurements in a vacuum.

Mechanical impedance, given by equation (13), is a complex equation; it can therefore be written as a function of its real part and its imaginary part:

\[ Z_{mc}(\omega) = K_m(\omega) + jX_m(\omega). \]  

(16)

Where, \( K_m(\omega) \) is the compression stiffness or the mechanical resistance to an angular frequency \( \omega \).

\( X_m(\omega) \) is the mechanical reactance\(^1\).
At low ranges, we assume that inertial effects are insignificant. Therefore, according to Langlois et al. (2001) we can obtain a close approximation of the reactance by multiplying the mechanical resistance by the material damping loss factor ($\eta$).

\[ Z_{mc}(\omega) \cong K_m(\omega)(1 + j\eta(\omega)). \tag{17} \]

In the quasi-static range, compression stiffness and loss factor are given by:

\[ K_m(\omega) = \text{Re}(Z_{mc}(\omega)), \tag{18} \]

and

\[ \eta(\omega) \cong \frac{\text{Im}(Z_{mc}(\omega))}{\text{Re}(Z_{mc}(\omega))}. \tag{19} \]

2.2.2.1 Polynomial ratios

Simulations of static finite elements:

For a long slim sample (normally $s < 0.025$, when $s$ is the shape factor), Poisson’s modulus and the surrounding conditions do not usually affect static compression stiffness and can be expressed as follows:

\[ K_0 = \frac{EA}{L}. \tag{20} \]

Where, $A$ is the transversal section of the sample. $L$ is the length of the sample.

But when the sample is short (normally $s > 0.025$), the effects of Poisson and the surrounding conditions have an effect. In this case, $K_m(0)$ is replaced by $K_0$ in equation (20) and isolating the apparent Young modulus ($E'$) we obtain equation (21).

\[ E' = \frac{L}{A} K_m(0). \tag{21} \]

With equations (13), (20) and (21) a poroelastic material can be modeled in finite elements.
So, introducing a Young modulus \((E)\) and Poisson’s modulus \((\nu)\) and with different shape factors \((s)\), we obtain different compression moduli \((K_0)\) using equation (20). On the other hand, we introduce a force \((F(\omega))\), in the samples and obtain different displacements \((u(\omega))\) for different shape factors. With the latter, we can define compression stiffness \((K_m(0))\) using equation (18). Dividing \(K_m(0)\) and \(K_0\) we obtain the normalized the static modulus of compression, which is independent from Young’s modulus. The results obtained by Langlois et al (2001) in the simulation are shown in Figure 5:

![Figure 5: Variations of static compression stiffness depending on shape factor and Poisson’s modulus. (Langlois et al, 2001).](image)

Each curve in Figure 5 can be written as an \(M\) order polynomial to create polynomial relations depending on the shape factor:

\[
P\nu(s) = \frac{K_m(0)}{K_0} = 1 + \sum_{i=1}^{M} C_i^{\nu} s^i .
\]  

(22)

Where, \(C_i^{\nu}\) are the coefficients of the curve referred to curve \(\nu\).

If, instead of expressing the polynomial relations dependent on the shape factor, they are expressed dependent on Poisson’s modulus (\(N\) order polynomial):
\[ P_i(\nu) = \frac{K_m(0)}{K_0} = 1 + \sum_{i=1}^{N} D_i \nu^i. \]  

(23)

Where, \( D_i \) are the coefficients of the curve referred to curve \( s \).

If working under quasi-static conditions, the following substitution can be performed in equations (22) and (23): \( K_m(\omega) \rightarrow K_m(0) \) for \( \omega << \omega_i \). Where, \( \omega_i \) is the first resonance of the system.

Determining Poisson’s modulus:

If we take two samples (samples 1 and 2) of the same material, Young’s modulus \( E \), Poisson’s modulus \( \nu \) and the damping factor \( \eta \) are identical in both samples and combining the equations (20), (21), (22) and (23) we obtain:

\[ E(\omega) = \frac{K_{m,s_i}(\omega)L_{s_i}}{P_{s_i}(\nu)A_{s_i}} = \frac{E'_s(\omega)}{P_{s_i}(\nu)} \quad i = 1,2. \]  

(24)

Where, \( K_{m,s_i}(\omega) \) and \( E'_s(\omega) \) are the stiffness compression measurements and the apparent Young modulus of the sample at an angular frequency of \( \omega \). \( L_{s_i} \) and \( A_{s_i} \) are the thickness and section of the samples \((i = 1,2)\).

In equation (25), the polynomial \( P_i \) can be taken as a correction factor to apply in the apparent Young modulus to obtain the true value of Young’s modulus.

Combining equation (24) for the case of two samples of the same material, we obtain the following equation:

\[ \frac{E'_s(\omega)}{P_{s_1}(\nu)} - \frac{E'_s(\omega)}{P_{s_2}(\nu)} = 0. \]  

(25)

As the apparent Young modulus is determined by the geometry and compression stiffnesses \((K_{m,s_1}(\omega) \text{ and } K_{m,s_2}(\omega))\), equation (25) has only one unknown: Poisson’s modulus.

Determining Young’s modulus:
Once Poisson’s modulus has been calculated, Young’s modulus is directly determined by equation (24).

**Determination of the loss factor.**

This parameter is estimated from the real and imaginary parts of the mechanical impedance (see equation (19)).

### 2.3 Parameters that define coupling between the fluid and skeleton

The parameters that define the coupling between the two phases that comprise the porous material are as follow: porosity, resistivity, tortuosity, and viscous and thermal lengths. Also described for each is the experimental method for obtaining quantification.

#### 2.3.1 Porosity

Materials such as fiber glass and polymer foams with open pores are elastic structures with surrounding air. Porosity, $\phi$, is the air volume quotient, $V_f$, and the total volume, $V_t$, of poroelastic material:

$$\phi = \frac{V_f}{V_t}. \quad (26)$$

In air volume, only the open pores are considered, and therefore the closed pores pertain to the volume of the elastic structure or skeleton.

The value of porosity ranges between 0 and 1. For absorbing materials (polymer foams and fibrous materials it is $0.95 < \phi < 0.99$).

According to Champoux et al (1990), taken from Dauchez (1999), measurement of porosity is based on the Boyle-Mariotte law for perfect gases, and consists in supposing that $pV = cte$. In the diagram in Figure 6, is shown how a porosimeter works. The piston produces a variation in pressure from which is deduced the air in the material sample.
where \( p_0 \), is the initial pressure and \( V_{\text{ext}} \) is the volume not occupied by the sample.

Therefore, applying Boyle-Mariotte’s law, we obtain:

\[
(V_f + V_{\text{ext}})p_0 = (V_f + V_{\text{ext}} + \Delta V)(p_0 + \Delta p),
\]

(27)

\[
V_f = -\left(\frac{p_0 + \Delta p}{\Delta p}\right) \left(\Delta V + V_{\text{ext}}\right).
\]

(28)

Once the air volume of the sample is obtained, porosity is determined:

\[
\phi = \frac{V_f}{V_s}.
\]

2.3.2 Resistivity

Resistivity, \( \sigma \), describes the viscous interactions dependent on frequency. Under a certain frequency, the thickness of the viscous layer is equal to

\[
\delta = \frac{2\eta}{\rho_0 \omega},
\]

(29)

which is much greater than the size of the pore. Therefore, the viscous friction forces act throughout the fluid domain. The forces of inertia tend to be annulled. The sound wave generated a pressure gradient opposed by the viscous friction forces.

Darcy’s law (Sánchez San Román, 2003) establishes that the flow, \( Q \), which circulated through a conduit divided by a porous medium is equal to a constant known as permeability, per section, \( S \), of the conduit and the hydraulic gradient (which is the difference in pressure, \( \Delta p \), per unit
of porous medium thickness, \( l \). Permeability presents a negative sign, since the flow is a vectorial magnitud in the direction of decreasing \( \Delta p \):

\[
Q = C_{\text{permeability}} \frac{\Delta p}{l}.
\]

This linear relation between the flow and the hydraulic gradient is not fulfilled for very low values of permeability (very high resistivity) or very high flow speed values. Beranek and Vér (1992) give a limit value of \( 0.5 \times 10^{-4} \text{ m/s} \), after which resistivity depends on speed.

Resistivity is the inverse of permeability as defined by Darcy and therefore equal to:

\[
\sigma = -\frac{\Delta p}{l} \frac{S}{Q}.
\]

Where, \( l \) is the thickness of the porous material, \( S \) is the section of the material and \( Q \) is the volumetric flow. The measuring units are rayls/m or, Ns/m\(^4\).

Intrinsic permeability of porous material is equal to \( k_0 = \eta/\sigma \) in m\(^2\).

For common absorbing materials \( 10^3 \text{ Nm}^{-4}\text{s} < \sigma < 10^6 \text{ Nm}^{-4}\text{s} \).

The measurement of air flow resistivity is performed in accordance with ISO 29053 standard (1991), that defines two means of measurement: by constant or variable flow.

- By constant flow.

The diagram of the resistometer for constant flow is shown in Figure 7.

![Diagram of the resistometer for constant flow. (Dauchez, 1999).](image)
Two porous materials are placed serially, as explained by Dauchez (1999). The first material is used as reference, since its resistance to air flow is known, $R_1$. We wish to determine the resistance to flow of the second material, $R_2$, and as the section $S$ and thickness $l$, are known, we can determine the resistivity to air flow by the ratio:

$$\sigma = R_2 \frac{S}{l}.$$  \hspace{1cm} (32)

To determine $R_2$, we know that for a constant flow the following must be fulfilled:

$$\frac{\Delta p_1}{R_1} = \frac{\Delta p_2}{R_2}.$$  \hspace{1cm} (33)

Therefore, resistivity to air flow will be:

$$\sigma = R_1 \frac{\Delta p_2 S}{\Delta p_1 l}.$$  \hspace{1cm} (34)

In this direct method of constant flow, a flow of stable air is passed through a test tube at a speed of $0.5 \cdot 10^{-3}$ m/s. The variation in pressure is measured with a manometer.

Generating the air flow at constant speed is complicated. If performed using a vacuum pump, the flow may be irregular and spaced, so the pump must provide a regular, smooth air flow. The same occurs if a compressor is used, except that in this case we have to ensure that the air is clean and the flow passing through the material is laminar. If performed with a water column, the column should be large enough to allow time for measurement before it empties and the height variation should not exert too much influence on the speed of water feed.

Achieving constant air flow and finding the measuring instruments are the major disadvantages of this method.

- By variable flow.

The diagram of the resistometer for variable flow is shown in Figure 8.
Figure 8: Diagram of the resistometer for variable flow. (Dauchez, 1999).

Figure 9: Resistometer for variable flow built in Ikerlan. (Mendibil, 2004).

As explained by Mendibil (2004), this method measures the pressure with respect to the balance pressure. Since the piston generates an oscillating movement, the air pressure inside the measuring cell increases and decreases at the same frequency as the piston, and is measured with a condenser microphone. This microphone has to be calibrated with a pistonphone running at 2 Hz.

Piston movement must be senoidal. This movement is achieved using a lever or a mechanical system. Recommended r.m.s. value of linear flow speed $u_{r.m.s.}$ should be between 0.5mm/s and 4mm/s.
\[ u_{\text{r.m.s.}} = \frac{\pi}{\sqrt{2}} f h \frac{A_p}{S_2}. \]  

(35)

Where \( f \) is piston frequency, \( h \) the peak-to-peak range of stroke, \( A_p \) piston surface area and \( S_2 \) the sample surface area. The flow generator has a variety of ranges, different from those of the pistonphone so it can work in the range of recommended speeds.

Therefore, resistivity will be:

\[ \sigma = \frac{1}{u_{\text{r.m.s.}}} \frac{\Delta p}{l}. \]  

(36)

The disadvantage of the alternate flow method is the need to interpret the signal given by the microphone. Filters are necessary to isolate the 2 Hz signal and measure it. In addition, the microphone will have to be capable of good response to infrasonic frequencies. Normally, microphones are used to work between ranges of 20 Hz to 20 kHz, which is the range of frequencies heard by the human ear.

2.3.3 Tortuosity

Tortuosity, also known as the structure shape factor, is a parameter that takes into account the irregular shape of the pore and the non-uniform distribution of pores per unit of section throughout the thickness of the poroelastic material.

The simplest way to calculate tortuosity is to suppose that the elastic material presents cylindrical pores at an angle of \( \varphi \) with respect to the direction of the thickness of the material.

![Figure 10: Elastic material with cylindrical pores at an angle of \( \varphi \).](image)
If \( n \) is the number of pores per unit of surface area, porosity will be defined as:

\[
\phi = \frac{n \pi r^2}{\cos \varphi}, \tag{37}
\]

where \( r \) is the radius of the pore. Poiseuille’s law, only valid in the laminar regimen, establishes that the pressure gradient, \( \Delta p \), in a circular tube with radius \( r \) is given by:

\[
\frac{\Delta p}{\Delta l} = -\frac{8\eta Q}{\pi r^4}, \tag{38}
\]

where \( Q \) is the volumetric flow in the conduit and \( \eta \) is the dynamic viscosity of the fluid.

If we now have a volumetric flow that acts on a material surface with inclined cylindrical pores, the flow per unit of surface area will be, \( Q' \). Therefore, in each of the cylindrical pores it will be:

\[
\frac{\Delta p}{\Delta l} = -\frac{8\eta Q'}{n \pi r^4}. \tag{39}
\]

On the other hand, we know that resistivity per flow per unit of surface area is equal to the pressure gradient, that is:

\[
-\frac{\Delta p}{Q'} = \sigma Q', \tag{40}
\]

or:

\[
-\frac{\Delta p}{\Delta l \cos \varphi} = \sigma Q'. \tag{41}
\]

Isolating \( \frac{\Delta p}{\Delta l \frac{1}{Q'}} \) from equations (39) and (41) we obtain,

\[
\sigma = \frac{8\eta}{n \pi r^2 \frac{1}{r^2} \cos \varphi}. \tag{42}
\]

Introducing the porosity value, we have:
\[ \sigma = \frac{8\eta}{\phi r^2 \cos^2 \varphi}. \quad (43) \]

However, Zwikker and Kosten (1949), taken from Fahy (2001), show us that in flow analysis in a cylindrical conduit with radius \( r \), a parameter exists that describes the changes in viscosity, whose value is:

\[ s = \left( \frac{\rho \omega r^2}{\eta} \right)^{\frac{1}{2}}. \quad (44) \]

Isolating \( r^2/\eta \) from equation (43) and substituting in equation (44) we obtain:

\[ s = \left( \frac{8\omega \rho k_s}{\sigma \phi} \right)^{\frac{1}{2}}. \quad (45) \]

Where \( k_s \) is the tortuosity and its value for inclined cylindrical pores is equal to:

\[ k_s = \frac{1}{\cos^2 \varphi}. \quad (46) \]

Dauchez (1999) demonstrated that tortuosity is a parameter that describes inertial coupling, and translated into an increase in volume mass of the fluid at macroscopic level. From the equality of kinetic energies at both microscopic and macroscopic level, we obtain:

\[ \frac{1}{2} \rho_0 \langle \dot{U}_m^2 \rangle = \frac{1}{2} \rho_0 k_s \dot{U}^2. \quad (47) \]

Where, \( \dot{U}_m \) is the microscopic speed of the fluid, \( \dot{U} = \langle \dot{U}_m \rangle \) is the macroscopic speed of the fluid, \( \rho_0 \) is fluid density and \( \langle \rangle \) indicates mean volume.

As the mean of the squares is greater than the square of the means, tortuosity is always greater than the unit.

Johnson et al (1987), taken from Allard (1993), affirm that tortuosity so defined, i.e., as:

\[ k_s = \frac{\langle \dot{U}_m^2 \rangle}{\dot{U}^2}, \quad (48) \]
is not only valid for cylindrical pores but also for all types of structure geometry.

Johnson et al. (1987), taken from Allard (1993) define effective density as $\rho = \alpha(\omega)\rho_0$, where $\alpha(\omega)$ is dynamic tortuosity. This tortuosity is equal to $k_s$ when $\omega \to \infty$. That is:

$$\alpha_\infty = k_s.$$  \hspace{1cm} (49)

As the nomenclature of $\alpha_\infty$ is more common in current literature, we will use it to refer to tortuosity hereafter.

Experimentally, there is an electrical method to determine tortuosity of a porous material. The method is based on the measurement of resistivity of a porous sample saturated with conductor fluid, following the equivalence between the field of microscopic speeds of the fluid and the current fields (Brown, 1980, taken from Dauchez, 1999).

The diagram of the tortuosity meter is shown in Figure 11.

Figure 11: Diagram of the tortuousity meter. (Dauchez, 1999).

Tortuosity is given by:

$$\alpha_\infty = \phi \frac{R_e}{R_{e0}}.$$ \hspace{1cm} (50)

Where $R_{e0}$ is resistivity measured without a porous sample and $R_e$ is resistivity with the porous sample.

The quotient between resistivities is equal to:
\[
\frac{R}{R_0} = 1 - \frac{l}{l} + \frac{U_c l_0}{U_c l}.
\] (51)

Where, \(U_c\) and \(U_c^0\) are the voltages in the electrodes.

Melon and Castagnéde (1995), taken from Dauchez, 1999, propose a method to determine tortuosity based on ultrasonic waves. The method is based on the delay, \(\Delta t(\omega)\), originated by an ultrasound wave passing through the material. This delay is obtained by comparing the time needed by the ultrasound wave to go from one pickup to another, depending on whether a sample of poroelastic material is introduced or not between the two pickups.

The refraction index, \(n(\omega)\), is defined as the quotient between the two sound speeds in an acoustic medium \((c_0)\) and a porous medium \((c(\omega))\):

\[
n(\omega) = \frac{c_0}{c(\omega)} = 1 + \frac{c_0 \Delta t(\omega)}{l}.
\] (52)

If this refraction index is squared, we obtain:
\[ n^2(\omega) = \alpha_n \left[ 1 + \delta \left( \frac{1}{\Lambda} + \frac{\gamma-1}{\sqrt{P \Lambda'}} \right) \right]. \] (53)

Where \( \delta \) is maximum viscous layer defined in equation (29) and \( \Lambda \) and \( \Lambda' \) are the characteristic viscosity and thermal lengths, respectively, defined in subsections (2.3.4 and 2.3.5).

If we represent the square of the refraction index as a function of the limit layer, the ordinate in origen is tortuosity.

This method, as stated by Ayrault (1999), taken from Dauchez (1999), in his thesis, is not valid in two situations:

- When material resistivity is high, i.e., when the ultrasound wave is greatly dampened.
- When wavelength is greater than the size of the pore.

Fellah et al. (2003) propose a faster, non-destructive method to measure tortuosity and porosity. These measurements are based on using ultrasound waves that incide obliquely on a porous sample and measuring the reflected wave for a variety of incidence angles. Figure 13 shows the diagram used by Fellah et al. (2003), where the initials P.G correspond to the pulsed generator, H.F.F-P.A. to the preamplifier and high frequency filter, D.O to the digital oscilloscope, C to the ordinate and S to the porous sample.

![Diagram](image)

Figure 13: Porosity and tortuosity measurement using ultrasound techniques. (Fellah, 2003).

These determine the tortuosity and porosity of the following equations:
\[
\alpha_* = \frac{\left(1-r_2\right)\left(1+r_1\right)\cos \theta_2}{\left(1+r_2\right)\left(1-r_1\right)\cos \theta_1} \sin^2 \theta_1 - \sin^2 \theta_2 \frac{\left(1-r_2\right)\left(1+r_1\right)\cos \theta_2}{\left(1+r_2\right)\left(1-r_1\right)\cos \theta_1} - 1 \tag{54}
\]

where \( \theta_1 \) and \( \theta_2 \) are the incidence angles, and \( r_1 \) and \( r_2 \) are the reflection coefficients for the respective angles.

\[
\phi = \frac{\alpha_* (1-r_1) \cos \theta_1}{\left(1+r_1\right)\sqrt{\alpha_* - \sin^2 \theta_1}} \tag{55}
\]

for \( i = 1, 2 \).

### 2.3.4 Viscous characteristic length

Viscous characteristic length, \( \Lambda \), describes the effects of viscosity at high frequencies. Over a certain frequency, the forces of inertia dominate the forces of viscosity, in which the effect of viscosity is produced only in the proximity of the skeleton walls.

This is associated with the flow regimen. In other words, when Reynolds number is equal to the quotient between inertial forces and viscous forces, at a certain frequency, inertial forces are much greater than viscous forces.

Viscous forces produce shear forces on the surface that are proportional to the speed gradient. At low frequencies, speed distribution goes from zero, at the conduit wall to a maximum on the interior of the wall. The region between both states is the limit layer. Therefore, we can define the limit layer as the region that separated the two states of minimum and maximum velocity.

In a cylindrical conduit with a radius of \( r \), at low frequencies, the limit layer, \( \delta \), is equal to the radius, while at high frequencies \( \delta < r \), as shown in Figure 14:
Dauchez (1999) establishes that the integration of viscous forces throughout the fluid domain $V_f$ depends on the relation $\delta / \Lambda$ with

$$\Lambda = 2 \int_{S_i} |\vec{U}_m|^2 dV \int_{S_i} |\vec{U}_m|^2 dS,$$  \hspace{1cm} (56)

where $S_i$ is the contact interface between the fluid and the skeleton.

Johnson et al. (1987), taken from Allard (1993) give an expression for dynamic tortuosity at high frequencies:

$$\lim \alpha(\omega) = \alpha_0 \left[1 + \left(1 - j \frac{\delta}{\Lambda}\right)\right].$$  \hspace{1cm} (57)

The behavior of $\alpha(\omega)$ depends on $\alpha_0$ and $\Lambda$ at high frequencies.

Panneton (1996), taken from Dauchez (1999), defines a transition frequency, $f_{T_v}$, between the viscous effects at low and high frequencies, governed by permeability $\left(k_0 = \eta / \sigma\right)$ and $\Lambda$ respectively:

$$f_{T_v} = \frac{\phi^2 \Lambda^2 \sigma^2}{8 \pi \eta \alpha_0^2 \rho_0}.$$  \hspace{1cm} (58)

### 2.3.5 Thermal characteristic length

Thermal characteristic length, $\Lambda'$, describes the thermal changes between the two phases, solid and fluid, at high frequencies. Since thermal inertia of the skeleton is greater than that of the fluid, the
modulus of compressibility of the fluid suffers a modification and takes on values between the isothermal compressibility modulus at low frequencies and the adiabatic modulus at high frequencies.

In an analogous manner to what occurs with the viscous effect, there is a frequency after which the thermal effects only occur in the proximity of the skeleton walls. According to Dauchez (1999) the integration of these effects throughout the dominion depends on the quotient $\delta'/\Lambda'$, where the thermal limit layer:

$$\delta' = \frac{\delta}{\sqrt{P_t}}.$$  \hfill (59)

Where $\delta'$ is the viscous limit layer and $P_t$ is Prandtl's number. And the thermal characteristic length:

$$\Lambda' = \frac{2V_f}{S_i}.$$  \hfill (60)

Where $V_f$ is the fluid volume and $S_i$ is the interface fluid-skeleton contact surface.

For common poroelastic materials, Dauchez (1999) estimates that $\Lambda'$ is between two and threefold $\Lambda$.

In recent studies, Lafarge et al. (1997) define static permeability, $k_0'$, and another dynamic thermal permeability. The first is similar to viscous permeability, $k_0$. Using static thermal permeability and thermal characteristic length, they construct a model to estimate dynamic thermal permeability $k'(\omega)$ analogous to that defined by Johnson et al. (1987) for viscous dynamic permeability.

Therefore, as an analogy, a transition frequency, $f_{Tr}$, can be defined between thermal effects at low and high frequencies, respectively governed by $k_0'$ and $\Lambda'$ so that:

$$f_{Tr} = \frac{\eta\phi^2 \Lambda'^2}{8\pi k_0'^2 \rho_0 P_t},$$  \hfill (61)

where $\rho_0$ and $\eta$ are the density and viscosity of the fluid, respectively, $\phi$ is the porosity and $P_t$ Prandtl's number.
However, Lafarge et al. (1997), in the conclusions of their work, conclude that due to the complexity of material structures, it was not possible to compare acoustically evaluated thermal static permeability with theoretical estimated values.

A physical-chemical method exists to measure thermal characteristic length, known as B.E.T., based on the measurement of gas molecules that cover the skeleton pores (Lemarinier et al (1995), taken from Dauchez, 1999). Currently, ultrasound wave measurements are used.

The thermal and viscous characteristic lengths are related with the square of the refraction index of equation (53), in such a way that, representing it as a function of the limit layer, its angle depends on these characteristic lengths.

3. CHARACTERIZATION MODELS OF POROELASTIC MATERIALS

The first models used to predict the behaviour of poroelastic models were based on empirical laws such as those developed by Delany and Bazley (1970), where rock wool surface impedance was estimated considering a rigid skeleton.

There are other models that assume the rigid skeleton hypothesis, since this porous medium skeleton presents high stiffness as regards fluid. These models are known as equivalent fluid models.

A more complete theoretical model that takes into consideration movements of both phases using numerous parameters that are characteristic of the skeleton and fluid can be found in the works of Biot (1956a, 1956b). Biot’s general theory is complex and predictive numerical models are often necessary.

The purpose of all the models is to acoustically characterize fibrous absorbing materials by constant complex wave propagation, $\Gamma_c = jk_c$ (where $k_c$ is the characteristic wave number of the material) and complex characteristic impedance, $Z_c$.

Establishing $Z_c$ and $\Gamma_c$ for a porous material allows complete characterization of its acoustic behaviour and to describe sound wave propagation in its interior.

$$Z_c = Z_c' + jZ_c^\prime$$  \hspace{1cm} (62)
\[ \Gamma_c = \Gamma'_c + j\Gamma''_c, \]  

(63)

where \(\Gamma'_c\) is the attenuation constant, \(\Gamma''_c\) is the phase exponent, \(Z'_c\) is the acoustic resistance and \(Z''_c\) is the acoustic reactance.

\(\Gamma'_c\) is the dampening constant. This parameter is easy to measure with a microphone sensor. Decrease in sound pressure is measured \((\text{nepers/m})^2\) in a plane sound wave propagated in a thick layer of material. \(\Gamma''_c\), is obtained measuring the phase change with distance. On the other hand, complex characteristic impedance is measured in an impedance tube.

A more generic way of seeing these two impedance variables and wave number is as a function of the compressibility modulus, density and angular frequency. This way, we obtain:

\[ Z_c = (K(\omega)\rho(\omega))^{1/2}, \]  

(64)

\[ k_c = \omega(\rho(\omega)/K(\omega))^{1/2}. \]  

(65)

The various models currently used attempt to determine the compressibility modulus and equivalent density for poroelastic material.

### 3.1 Empirical models

Empirical models estimate the impedance and complex propagation constant of material through knowledge of the secondary properties of the material. Following we describe the Delaney-Bazley model, that provides these values as a function of flow resistivity, the Mechel-Vér model and the Allard-Champoux model, both of which are corrections to the Delaney-Bazley model.

#### 3.1.1 Delany-Bazley model

Delany and Bazley (1970) derived the empirical model that estimates the value of the propagation constant, \(jk_c\), and characteristic impedance, \(Z_c\), as a function of material flow resistivity.

After numerous measurements using Kundt’s tube or impedance tube for materials of varying resistivity and a specific frequency range, Delany and Bazley established the laws by which
impedance and the damping constant are estimated as a function of the quotient between frequency and flow resistivity.

These laws are described in their work (1970) as follows (see the graphs and equations or laws for each, where $R$ and $X$ are the real and imaginary part of $Z_C$ respectively; $\alpha$ and $\beta$ are the real and imaginary part of $\Gamma_C$ respectively):

![Figure 15: Normalization of the real part (a) and the imaginary part (b) of characteristic impedance as a function of the quotient between frequency and flow resistivity. (Delany and Bazley, 1970).](image)

![Figure 16: Normalization of the real part (a) and the imaginary part (b) of the attenuation constant as a function of the quotient between frequency and flow resistivity. (Delany and Bazley, 1970).](image)

These laws can be written as a function of an adimensional parameter that multiplies the density of the fluid by the quotient between frequency and resistivity, $\rho_0 f / \sigma$, that Delany and Bazley
called the normalized adimensional parameter. Allard (1993) uses the Delany-Bazley laws as a function of this parameter:

\[
Z_c = \rho_0c\left[1 + 0.0571\left(\frac{\rho_0f}{\sigma}\right)^{-0.754} - j0.087\left(\frac{\rho_0f}{\sigma}\right)^{-0.732}\right],
\]

\[
k_c = \frac{\omega}{c}\left[1 + 0.0978\left(\frac{\rho_0f}{\sigma}\right)^{-0.7} - j0.189\left(\frac{\rho_0f}{\sigma}\right)^{-0.595}\right].
\]

The model assumes that the absorbing material is fibrous and that the fibers are uniformly distributed. Delaney and Bazley establish a range of validity for the empirical curves obtained:

\[10 \leq f/\sigma \leq 1000,\]

where frequency, \(f\), is expressed in Hz and air flow resistivity, \(\sigma\), in gr/cm\(^3\)s. So a validity range can be established as a function of the normalized adimensional parameter:

\[0.01 \leq \rho_0f/\sigma \leq 1.\]

It must be borne in mind that all the fibrous materials considered in this model present porosity close to one. Delaney and Bazley explain that another normalization procedure should be performed for materials with lower porosity.

### 3.1.2 Mechel-Ver model

In Chapter 8, Beranek and Vér (1992) mention that the Mechel and Vér model is a more refined adjustment than the Delany-Bazley model. It differentiates two families of absorbing materials and two areas for the normalized adimensional parameter, that they call the normalized frequency parameter, \(E = \rho_0f/\sigma\).

\[
Z_c = \rho_0c\left(1 + b'E^{-\beta} - jb^*E^{-\beta}\right),
\]

\[
\Gamma_c = \frac{\omega}{c}\left[a'E^{-\alpha} + j\left(1 + a^*E^{-\alpha}\right)\right].
\]

That is,
The values of the coefficients and exponents are summed up in Table 2: The values of the coefficients and exponents of the Mechel-Vér model. (Beranek and Vér, 1992). The Mechel-Vér model is more precise at low frequencies than the Delany-Bazley model.

<table>
<thead>
<tr>
<th>Material Region</th>
<th>$a'$</th>
<th>$a''$</th>
<th>$b'$</th>
<th>$b''$</th>
<th>$\beta'$</th>
<th>$\beta''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E \leq 0.025$</td>
<td>0.322</td>
<td>0.502</td>
<td>0.136</td>
<td>0.641</td>
<td>0.081</td>
<td>0.699</td>
</tr>
<tr>
<td>$E &gt; 0.025$</td>
<td>0.179</td>
<td>0.663</td>
<td>0.103</td>
<td>0.716</td>
<td>0.0563</td>
<td>0.725</td>
</tr>
<tr>
<td>$E \leq 0.025$</td>
<td>0.396</td>
<td>0.458</td>
<td>0.135</td>
<td>0.646</td>
<td>0.0668</td>
<td>0.707</td>
</tr>
<tr>
<td>$E &gt; 0.025$</td>
<td>0.179</td>
<td>0.674</td>
<td>0.102</td>
<td>0.705</td>
<td>0.0235</td>
<td>0.887</td>
</tr>
</tbody>
</table>

Table 2: The values of the coefficients and exponents of the Mechel-Vér model. (Beranek and Vér, 1992).

### 3.1.3 Allard-Champoux model

One of the most recent empirical models is that of Allard and Champoux (1992), taken from Prieto and Bermúdez (2003). This model assumes that the thermal effects depend on the frequency. According to Tikander (2002) the equations by Allard and Champoux show better results at low frequencies that the previous models.

The Allard-Champoux model (1992) defines dynamic density and the compressibility modulus as follows:

\[
\rho(\omega) = \rho_0 \left[ 1 - j \left( \frac{\sigma}{\rho_0 \omega} \right) G_1 \left( \frac{\rho_0 \omega}{\sigma} \right) \right], \tag{71}
\]

\[
K(\omega) = \gamma \rho_0 \left[ \frac{\gamma - 1}{1 - \left( \frac{j}{4P_r} \right) \frac{\sigma}{\rho_0 \omega} G_2 \left( \frac{\rho_0 \omega}{\sigma} \right)} \right]^{-1}. \tag{72}
\]

Where the functions $G_1 \left( \frac{\rho_0 \omega}{\sigma} \right)$ and $G_2 \left( \frac{\rho_0 \omega}{\sigma} \right)$ are:
\[ G_1\left( \frac{\rho_0 \omega}{\sigma} \right) = \sqrt{1 + \frac{j}{2} \left( \frac{\rho_0 \omega}{\sigma} \right)}, \quad (73) \]

\[ G_2\left( \frac{\rho_0 \omega}{\sigma} \right) = G_1\left( \frac{\rho_0 \omega}{\sigma} \right) \left( 4P_r \left( \frac{\rho_0 \omega}{\sigma} \right) \right). \quad (74) \]

Where, \( P_0 \) is the balance pressure.

These empirical models are used to estimate with good precision characteristic impedance and the material propagation coefficient. In Figure 17, Mendibil (2004) compares the models of Mechel, Delany-Bazley and Allard-Champoux with measurements taken in the impedance tube of a wool type material.

The empirical models estimate acoustic impedance and complex propagation constant values that are characteristic of the material. In the measurement with an impedance tube, the material is usually placed against a rigid wall or with a layer of air between the material and the wall. The thickness of the sample is normally small.
To know the impedance of finite thickness material in front of a rigid wall, knowing the values estimated with the models and to be able to compare it to the measurement in the Kundt tube, we use the following equation:

\[ Z_1 = Z_c \coth(jk_c d). \]  \hspace{1cm} (75)

Where \( d \) is the thickness of the sample.

### 3.2 Models of cylindrical pore poroelastic materials

Due to the complex geometry of pores in poroelastic materials, the first models that studied sound propagation assumed that the geometry was cylindrical.

In the study of sound propagation in poroelastic materials, the basic effects to study and analyze are viscosity and thermal. It is important to study these effects in poroelastic materials with cylindrical geometry.

Allard (1993) states that:

“The Kirchhoff theory (1868) of sound propagation in cylindrical tubes provides a general description of viscous and thermal effects, but this description is unnecessarily complicated for many applications. Moreover, the fundamental equations of acoustics that are used in the Kirchhoff theory can be very difficult to solve in the case of a non-circular cross-sections (page 48).”

Zwikker and Kosten (1949), taken from Allard (1993), completed a simplified model where they independently treated the effects of viscosity and thermal effects for circular section pores. According to Allard, based on other authors, the validity of this model is restricted to a pore radius range between 0.001 cm to several centimeters in acoustic frequencies.

Shown below are the effects of viscosity in a tube with a circular section following the Zwikker-Kosten model and the thermal effects are explained following Stinson’s model (1991), taken from Allard (1993).
3.2.1 Viscosity effect

Due to viscosity, the air suffers shear stress parallel to axis \( x_3 \) that is proportional to the variation in speed as regards \( x_1 \).

![Diagram showing the effects of viscosity in a tube with a circular section following the Zwikker-Kosten model.](image)

\[ \Delta F_3 = -\eta \frac{\partial v_3(x_1)}{\partial x_1} + \eta \frac{\partial v_3(x_1 + \Delta x_1)}{\partial x_1}. \]  

(76)

Where \( \eta \) is the dynamic viscosity and \( v_3 \) is the fluid speed in the direction of \( x_3 \).

The result of a force due to shear stress for a unit area will be:

\[ \Delta F_3 = -\eta \frac{\partial v_3(x_1)}{\partial x_1} + \eta \frac{\partial v_3(x_1 + \Delta x_1)}{\partial x_1}. \]  

(76)

Where \( \eta \) is the dynamic viscosity and \( v_3 \) is the fluid speed in the direction of \( x_3 \).

Force per unit of air volume and assuming that speed generally depends on \( x_1 \) and \( x_2 \), will be:

\[ X_3 = \eta \left( \frac{\partial^2 v_3}{\partial x_1^2} + \frac{\partial^2 v_3}{\partial x_2^2} \right). \]  

(77)

Using Newton’s second law, we have:

\[ j \omega \rho_0 v_3 = -\frac{\partial p}{\partial x_3} + \eta \left( \frac{\partial^2 v_3}{\partial x_1^2} + \frac{\partial^2 v_3}{\partial x_2^2} \right). \]  

(78)

As the problem is axis-symmetric, we can write Newton’s law as follows:
\[
\begin{align*}
    j \omega \rho_0 v_3 &= - \frac{\partial p}{\partial x_3} + \frac{\eta}{r \frac{\partial}{\partial r}} \left( r \frac{\partial v_3}{\partial r} \right). \\
    \text{where} & \\
    l &= \left(- \frac{j \omega \rho_0}{\eta} \right)^{1/2}, \quad (81)
\end{align*}
\]

Applying the surrounding condition that velocity must be null at surface \( r = R \) of the cylinder, we obtain the following solution to equation (79):

\[
\begin{align*}
v_3 &= - \frac{1}{j \omega \rho_0} \frac{\partial p}{\partial x_3} \left( 1 - \frac{J_0(lr)}{J_0(lR)} \right). \\
    \text{where} & \\
    & \ \ \ J_0 \text{is Bessel's zero order function.}
\end{align*}
\]

Obtaining mean velocity, \( v_3 \) as:

\[
\begin{align*}
v_3 &= \frac{\int_0^R v_3 \ 2\pi dr}{\pi R^2}, \\
    \text{and making use of the equivalence between Bessel's zero order and first order functions:} & \\
    \int_0^r r J_0(r) dr = a J_1(a),
\end{align*}
\]

we obtain the following expression for mean velocity:

\[
\begin{align*}
v_3 &= - \frac{1}{j \omega \rho_0} \frac{\partial p}{\partial x_3} \left( 1 - \frac{2}{s - j} \frac{J_1(s - j)}{J_0(s - j)} \right), \\
    \text{where} & \\
    s &= \left( \frac{\omega \rho_0 R^2}{\eta} \right)^{1/2}. \quad (85)
\end{align*}
\]

So, the equation (84) can be written as follows:
\[ j \omega \rho \mathbf{v}_3 = -\frac{\partial p}{\partial x_3}, \]  

(86)

obtaining an effective density equal to:

\[ \rho = \frac{\rho_0}{1 - 2 \frac{J_1(s\sqrt{-s})}{s\sqrt{-s} J_0(s\sqrt{-s})}}. \]  

(87)

On the other hand, density \( \rho_0 \) can be used in Newton's equation as:

\[ -\frac{\partial p}{\partial x_3} = j \omega \rho_0 \mathbf{v}_3 + \frac{2}{s\sqrt{-s} - j} \frac{J_1(s\sqrt{-s})}{J_0(s\sqrt{-s})} \frac{\rho_0 j \omega \mathbf{v}_3}{1 - 2 \frac{J_1(s\sqrt{-s})}{s\sqrt{-s} J_0(s\sqrt{-s})}}. \]  

(88)

3.2.2 Thermal effects

The thermal changes between the air and the structure modify the compressibility modulus of the air in the tubes. Stinson (1991), taken from Allard (1993), associates viscous and thermal effects to calculate this modulus, \( K \).

The linearized equation that describes thermal conduction in air is:

\[ \kappa \nabla^2 \tau = \frac{j \omega T_0}{p_0} (\rho_0 C_v \tau - p_0 C_p \xi). \]  

(89)

Where \( \kappa \) is thermal conductivity, \( C_v \) and \( C_p \) are specific heats at constant volume and pressure, respectively, \( p_0 \), \( T_0 \) and \( \rho_0 \) are the pressure, temperature and density of reference, respectively, and \( \tau \) and \( \xi \) are, respectively, the variations in temperature and density compared to the reference.

On the other hand, the equation of the air status, considered as an ideal gas, is:

\[ p = \frac{p_0}{p_0 T_0} (\rho_0 \tau + T_0 \xi). \]  

(90)

Eliminating \( \xi \) from equations (89) and (90) and making use of
\[ \rho_0 (C_p - C_v) = \frac{P_0}{T_0}, \]  
\[ (91) \]

we obtain the following equation:

\[ \nabla^2 \tau - \frac{j \omega \gamma}{\nu'} \tau = -j \frac{\omega}{\kappa} p, \]  
\[ (92) \]

where \( \gamma \) is the relation of specific heats and \( \nu' \) is:

\[ \nu' = \frac{\kappa}{\rho_0 C_v}. \]  
\[ (93) \]

Due to the fact that variations of \( \tau \) in the axial direction of the tube (\( x_3 \)) are smaller than in the cross-section direction (\( x_1 \) and \( x_2 \)) and defining an angular frequency \( \omega' \) equal to:

\[ \omega' = \omega \frac{\eta \gamma}{\rho_0 \nu'}, \]  
\[ (94) \]

we obtain a differential equation very similar to equation (78):

\[ \left( \frac{\partial^2 \tau}{\partial x_1^2} + \frac{\partial^2 \tau}{\partial x_2^2} \right) - j \omega' \rho_0 \tau \frac{\rho_0}{\eta} = -j \omega' \nu' \rho_0 p \frac{\rho_0}{\kappa \eta \gamma}. \]  
\[ (95) \]

The quantity expressed by \( \eta \gamma / \rho_0 \nu' \) is equal to Prandtl's number. Some authors such as Allard express this number as \( B^2 \).

Where the compressibility modulus is the product of density and pressure variation with respect to density, if this modulus is expressed as a function of mean density, \( \langle \xi^2 \rangle \)

\[ K = \frac{\rho_0 p}{\langle \xi \rangle}. \]  
\[ (96) \]

From equation (90) we obtain that mean density is equal to:

\[ \langle \xi \rangle = \frac{\rho_0}{P_0} p - \frac{\rho_0}{T_0} \langle \tau \rangle. \]  
\[ (97) \]
This way, analogously solving the differential equation for temperature, \( \tau \), we obtain the following expression for the compressibility modulus:

\[
K = \frac{\gamma P_0}{1 + (\gamma - 1) \frac{2}{B_s \sqrt{-j}} \frac{J_0(B_s \sqrt{-j})}{J_1(B_s \sqrt{-j})}}.
\]  (98)

### 3.3 Phenomenological models of equivalent fluid

In the previous section, 3.2, we have been able to study sound propagation in porous materials, taking into consideration cylindrical geometry. However, pore shapes are very complex, and it is nearly impossible to have exact models. This is why it is said that the models that study sound propagation through porous materials are phenomenological models.

The phenomenological models of equivalent fluid are those models that assume that the structure or skeleton of the porous material is rigid.

Allard (1993) offers two equivalent formulations for effective density and the compressibility model. The first formulation is expressed as a function of Johnson et al. (1987), \( G_1(\omega) \) and \( G_1'(B^2 \omega) \). These functions are simpler and easier to calculate than functions \( G_c(s) \) and \( G_c(Bs') \).

#### 3.3.1 First formulation

\[
\rho = \alpha_v \rho_0 \left[ 1 + \frac{\sigma \phi}{j \omega \rho_0} G_1(\omega) \right],
\]  (99)

\[
K = \frac{\gamma P_0}{\gamma - (\gamma - 1) \left[ 1 + \frac{\sigma' \phi}{j B^2 \omega \rho_0} G_1'(B^2 \omega) \right]}.
\]  (100)

Where,

\[
G_1(\omega) = \left( 1 + \frac{4 j \alpha^2 \eta \rho_0 \omega}{\sigma^2 \Lambda^2 \phi^2} \right)^{1/2},
\]  (101)
\[ G_j(B^2\omega) = \left(1 + \frac{4 j \alpha_x^2 \eta \rho_0 \omega B^2}{\sigma^2 \Lambda^2 \phi^2} \right)^{1/2}, \] (102)

\[ \Lambda = \frac{1}{c} \left( \frac{8\alpha_x \eta}{\phi \sigma} \right)^{1/2}, \] (103)

\[ \Lambda' = \frac{1}{c'} \left( \frac{8\alpha_x \eta}{\phi \sigma} \right)^{1/2} = \left( \frac{8\alpha_x \eta}{\phi \sigma'} \right)^{1/2}. \] (104)

### 3.3.2 Second formulation

\[ \rho = \alpha_x \rho_0 \left[ 1 + \frac{\sigma \phi}{j \omega \rho_0 \alpha_e} G_c(s) \right], \] (105)

\[ K = \frac{\gamma P_0}{\gamma - (\gamma - 1) \left[ 1 + \frac{\sigma' \phi}{j B^2 \omega \rho_0 \alpha_e} G_c'(B\omega) \right]^{-1}}. \] (106)

Where,

\[ G_c(s) = \frac{s \sqrt{-j} J_1(s \sqrt{-j})}{4} \left[ \frac{J_1(s \sqrt{-j})}{J_0(s \sqrt{-j})} \right]^{-1}, \] (107)

\[ s = c \left( \frac{8\omega \alpha_x \rho_0}{\phi \sigma} \right)^{1/2}, \] (108)

\[ s' = \left( \frac{8\omega \alpha_x \rho_0}{\phi \sigma'} \right)^{1/2} = \frac{1}{c'} \left( \frac{8\omega \alpha_x \rho_0}{\phi \sigma} \right)^{1/2}, \] (109)

where \( P_0 \) and \( \rho_0 \) are the reference pressure and density, respectively, \( \omega \) is the angular frequency, \( \eta \) is the fluid dynamic viscosity, \( B^2 \) is Prandtl's number, \( \gamma \) is the relation of specific heats, \( \phi \) is the
porosity, $\sigma$ is air flow resistivity, $\alpha_x$ is the tortuosity, $c$ is a parameter that depends on pore geometry (cross-section of the pore). $c'$ is a parameter analogous to $c$; $c'$ is always smaller than or equal to $c$ since $\Lambda$ is smaller than or equal to $\Lambda'$. For a circular section, parameter $c$ is equal to one.

### 3.3.3 Simple models

#### 3.3.3.1 Attenborough model

The expressions for effective density and the compressibility modulus given by Attenborough (1987), taken from Allard (1993) are as follows:

$$\rho = \rho_0 \alpha_x \frac{1}{1 - \frac{2}{\gamma - 1} \frac{J_1(s \sqrt{-j})}{J_0(s \sqrt{-j})}}. \quad (110)$$

$$K = \frac{\gamma \rho_0}{\gamma - (\gamma - 1) \frac{2}{B s \sqrt{-j} \sqrt{J_0(B s \sqrt{-j})}}} \quad (111)^3$$

Where,

$$s = b \left( \frac{8 \omega \alpha_x \rho_0}{\phi \sigma} \right)^{1/2}. \quad (112)$$

Expressions (110) and (111) are expressions of density and the compressibility modulus for a cylinder with a circular section with a radius of $R$

$$R = s \left( \frac{\eta}{\omega \rho_0} \right)^{1/2} = b \left( \frac{8 \alpha_x \eta}{\phi \sigma} \right)^{1/2}. \quad (113)$$

If the cylinder presents an inclined angle $\varphi$ with the direction of propagation, tortuosity is equal to:

$$\alpha_x = 1 / \cos^2 \varphi. \quad (114)$$

The two characteristic lengths are equal to the radius.
At low frequencies, the following approximation can be made:

\[
\rho = \rho_0 \alpha_s \left( 1 + \frac{8 j}{s^2} + \frac{1}{3} \right),
\]  

(115)

substituting the value of \( s^2 \)

\[
\rho = \rho_0 \alpha_s \left( 1 + \frac{1}{3} \right) + \frac{\phi \sigma}{j b^2 \omega}.
\]  

(116)

Considering the expression of effective density above, we observe that the model predicts resistivity equal to \( \sigma / b^2 \) instead of \( \sigma \). Therefore, the model is adequate when \( b \) is close to one.

At high frequencies, the model is valid when the characteristic lengths are similar to each other \( (\Lambda \cong \Lambda') \).

3.3.3.2 Allard simple model


Effective density and the compressibility modulus of this model are given by equations (105) and (106) respectively, but using \( s \) instead of \( s' \) in the calculation of the compressibility modulus.

The characteristic lengths in the model are:

\[
\Lambda = \frac{1}{b^2} \left( \frac{\eta}{\rho_0 \omega} \right)^{1/2} = \frac{1}{b} \left( \frac{8 \alpha_s \eta}{\phi \sigma} \right)^{1/2},
\]  

(117)

\[
\Lambda' = b \left( \frac{8 \alpha_s \eta}{\phi \sigma} \right)^{1/2}.
\]  

(118)

The low frequency limit is, \( \omega \to 0 \):

\[
\rho = \alpha_s \rho_0 \left( 1 + \frac{b^2}{3} \right) + \frac{\phi \sigma}{j \omega}.
\]  

(119)

As stated by the author, this model can be used when \( c' = 1/c = 1/b \).
3.4 Poroelastic phenomenological models

3.4.1 Biot model (taken from Allard, 1993)

Based on the work by Terzaghi (1923), taken from Schanz (2003), Biot presents a theoretical description of porous materials saturated with a viscous fluid (Biot, 1941; taken from Schanz, 2003). This was the starting point of Biot’s theory of poroelasticity. In the following years, Biot expanded this theory for cases of anisotropic materials (Biot, 1955, taken from Schanz, 2003) as well as for viscoelastic porous materials (Biot, 1956a). Biot’s dynamic theory was published in 1956 in two articles. The first article includes the research done in the low frequency range (Biot, 1956b), while the second covers the high frequency range (Biot 1956c). One of the most significant results of Biot’s theory is the identification of three types of waves for continuous material in 3D: two compression waves and one shear wave.

Biot’s theory considers completely saturated materials, while Vardoulakis and Beskos (1986) present an extension to materials partially saturated in fluid (such as water in sand, oil in rock or air in foams).

Biot’s phenomenological model attempts to predict the behaviour of sound propagation in materials with a flexible skeleton. In practice, these porous materials have elastic skeletons that are capable of transmitting sound waves; therefore, displacement of the solid part is not insignificant.

Tension and deformation in porous materials

The trajectory of a molecule of fluid that passes through a layer of porous material is complicated. Therefore, it is generally not easy to describe the microscopic movements. Given this problem, it is more common to analyze macroscopic movements and deformations.

If we assume that the porous medium is macroscopically homogeneous and isotropic, the pores are uniformly distributed in such a way that the properties of the material are the same in all directions.

The displacement vectors for the skeleton and fluid are \( \mathbf{u}^s \) and \( \mathbf{u}^f \) respectively, while the corresponding deformations tensors are represented as \( \varepsilon^s_\theta \) and \( \varepsilon^f_\theta \). So, the constitutive equations defining the tensions are:
\[
\sigma_y^s = [(P - 2N)\theta^s + Q\theta^f]\delta_y + 2Ne_y^s, \quad (120)
\]
\[
\sigma_y^f = (-\phi p) = Q\theta^s + R\theta^f. \quad (121)
\]

Where \( \delta_y \) is Kronecker’s delta:
\[
\delta_y = 1 \text{ if } i = j, \quad (122)
\]
\[
\delta_y = 0 \text{ if } i \neq j.
\]

\( \phi \) and \( p \) are porosity and pressure, respectively. \( \theta^s \) and \( \theta^f \) are skeleton and fluid dilatations, respectively. \( N \), \( R \), \( P \) and \( Q \) are the scalar coefficients defined by Biot.

If \( Q = 0 \), equation (120) is converted in the tension-deformation ratio of elastic solids. Therefore, the two terms, \( Q\theta^f \) and \( Q\theta^s \), show the relation between air dilatation and the tension that originates in the skeleton and vice versa, that is, the relation between skeleton dilatation and pressure variation generated in pore fluid; or, in other words, coupling between fluid and skeleton. In conclusion, we can say that Biot’s coefficient \( Q \) is the coefficient of coupling between the skeleton and the fluid of the pores.

To define or calculate coefficients \( N \), \( R \), \( P \) and \( Q \) that appear in equations (120) and (121) Biot defines three ‘Gedanken Experiments’ or experiments that give an idea as to how to measure these coefficients.

**Experiments to obtain \( P \), \( Q \), \( N \) and \( R \).**

Three experiments are performed to obtain these parameters.

1. In the first experiment, shear force is applied to the material. Dilatations generated in both the skeleton and the air are \( \theta^s = \theta^f = 0 \), therefore, equations (120) and (121) can be expressed as follows:
\[
\sigma_y^s = 2Ne_y^s, \quad (123)
\]
\[ \sigma_y^f = 0 . \]  

(124)

From equation (123) we deduce that \( N \) is the shear model since the fluid does not contribute to the shear force.

2. In the second experiment, the material is covered with a flexible case and hydrostatic pressure \( p_1 \) is applied. The air pressure inside the case remains constant and equal to \( p_0 \).

![Figure 19: The skeleton of the material is covered with a flexible case and submitted to hydrostatic pressure \( p_1 \) while the air inside remains constant at \( p_0 \).](image)

This experiment provides the definition of the compressibility modulus of the porous medium:

\[ K_b = -\frac{p_1}{\theta^i} \]  

(125)

Where, \( \theta^i \) represents skeleton dilatation.

Considering that for hydrostatic pressure compression, we have the following equalities

\[ \sigma_{11}^s = \sigma_{22}^s = \sigma_{33}^s = -p_1, \quad \sigma^s = e_{11}^s + e_{22}^s + e_{33}^s \quad \text{and} \quad e_{11}^s = e_{22}^s = e_{33}^s = \sigma^s/3, \]

for the configuration of Figure 19 equations (120) and (121) can be expressed as follows:

\[ -p_1 = \left( P - \frac{4}{3} N \right) \theta^s + Q \theta^f, \]  

(126)

\[ 0 = Q \theta^s + R \theta^f. \]  

(127)
Where, $\theta^i$ is the dilatation of the air in the material.

3. In the third experiment, represented in Figure 20, the material that is not covered is submitted to a pressure increase of $p_i$ in the fluid. This pressure variation is transmitted to the skeleton and, therefore, the equation of skeleton tensions becomes:

$$\sigma^s_{ij} = -p_i(1-\phi)\delta_{ij}. \quad (128)$$

Equations (120) and (121) can be expressed again as:

$$-p_i(1-\phi) = \left(P - \frac{4}{3}N\right)\theta^s_2 + Q\theta^f_2, \quad (129)$$

$$-\phi p_i = Q\theta^s_2 + R\theta^f_2. \quad (130)$$

Where, $\theta^s_2$ and $\theta^f_2$ are the dilatations of the skeleton and the air, respectively.

In this third experiment, the compressibility modulus of the skeleton (or the solid) is defined as:

$$K_i = -\frac{p_i}{\theta^s_2}. \quad (131)$$

It should be noted that in this last experiment, the material does not present changes in porosity, skeleton deformation is the same as if the material were not porous and it can be associated with a simple change of scale.

![Figure 20: Material submitted to pressure increase.](image)

The quantity $-p_i/\theta^f_2$ is the compressibility modulus of the fluid:
\[ K_i = -\frac{p_i}{\theta_i^2} \]  \hspace{1cm} (132)

Operating mathematically on equations (125) to (132), we obtain the following system of equations with three unknowns: \( P \), \( Q \) and \( R \).

\[ \frac{Q}{K_s} + \frac{R}{K_t} = \phi \]  \hspace{1cm} (133)

\[ \left( P - \frac{4}{3} N \right) \frac{1}{K_s} + \frac{Q}{K_t} = 1 - \phi \]  \hspace{1cm} (134)

\[ \left[ \left( P - \frac{4}{3} N \right) - \frac{Q^2}{R} \right] \frac{1}{K_b} = 1 \]  \hspace{1cm} (135)

Resolving the system, parameters \( P \), \( Q \) and \( R \) can be expressed as follows:

\[ P = \frac{(1 - \phi) \left[ 1 - \phi - \frac{K_b}{K_s} \right] K_s + \phi \frac{K_s}{K_t} K_b}{1 - \phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_t}} + \frac{4}{3} N \]  \hspace{1cm} (136)

\[ Q = \frac{\left[ 1 - \phi - \frac{K_b}{K_s} \right] \phi K_s}{1 - \phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_t}} \]  \hspace{1cm} (137)

\[ R = \frac{\phi^2 K_s}{1 - \phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_t}} \]  \hspace{1cm} (138)

If we apply the following hypotheses on the porosity, \( \phi \) and on the solid compressibility modulus, \( K_s \) and on the porous medium \( K_b \):

\[ \phi \approx 1, \quad K_b \ll K_s \]  \hspace{1cm} (139)

we obtain the following relations for coefficients \( P \), \( Q \) and \( R \).
In the case of poroelastic materials, the compressibility modulus $K_b$ can be evaluated as,

$$K_b = \frac{2N(N+1)}{3(1-2\nu)}.$$  \hfill (143)

While $K_f$ represents the compressibility modulus of the air in the material, that is, the compressibility modulus denominated by $K$ until now.

**Wave equations**

The tension-deformation relation of an isotropic elastic medium is expressed as seen in equation (144):

$$\sigma_{ij} = \lambda \delta_{ij} \delta + 2\mu \epsilon_{ij}.$$ \hfill (144)

Where,

$\delta_{ij}$ is Kronecker's delta (see equation (122)).

$\lambda$ and $\mu$ are Lamé's coefficients.

$E$, $\nu$ and $G$ are Young's modulus, Poisson's modulus and the shear modulus, respectively.

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}.$$ \hfill (145)

$$\mu = \frac{E}{2(1+\nu)} = G.$$ \hfill (146)

Taking into consideration that cubic dilatation, $\theta$, is expressed as follows:
\[ \theta = \nabla \cdot \mathbf{u} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = e_{11} + e_{22} + e_{33} . \]  

(147)

Where \( \mathbf{u} \) is the displacement vector.

The equation for the conservation of the linear moment is:

\[ \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} + \text{div} \sigma + X_i = 0 . \]  

(148)

Where \( X_i \) are the volume forces.

Substituting equation (144) in equation (148), we obtain the equation of movement of an elastic solid:

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = \lambda \frac{\partial \theta}{\partial x_i} + 2\mu \frac{\partial e_{ii}}{\partial x_i} + \sum_{j\neq i} 2\mu \frac{\partial e_{ji}}{\partial x_j} + X_i \quad i = 1,2,3 . \]  

(149)

Substituting \( e_{ii} \) for \( 1/2 \left( \partial u_j / \partial x_i + \partial u_i / \partial x_j \right) \) in the equation (149) we obtain the following movement equation for an elastic solid:

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = \lambda + \mu \frac{\partial \nabla \cdot \mathbf{u}}{\partial x_i} + \nu \nabla^2 u_i + X_i \quad i = 1,2,3 . \]  

(150)

Where \( \nabla^2 \) represents the Laplacian operator: \( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \). Equation (150) represents the movement equation of an elastic solid.

Biot’s model analyzes the behavior of porous materials. Starting with the equation of conservation of linear movement (148) and knowing that in porous material the tension tensor is equation (120), we obtain the movement equation of a porous material, for both its solid phase, equation (151), and its liquid phase, equation (152).

\[ (\rho_e + \rho_s) \frac{\partial^2 u_i^e}{\partial t^2} - \rho_s \frac{\partial^2 u_i^l}{\partial t^2} = (P - N) \frac{\partial \theta^e}{\partial x_i} + \nabla^2 u_i^e + Q \frac{\partial \theta^l}{\partial x_i} + X_i^l , \]  

(151)
\[ \phi \rho_0 + \phi \rho_a \left( \frac{\partial^2 u_i^f}{\partial t^2} - \rho_a \frac{\partial^2 u_i^s}{\partial t^2} \right) = R \frac{\partial \theta^f}{\partial x_i} + Q \frac{\partial \theta^s}{\partial x_i} + X_{1i}^i, \quad (152) \]

\[ i = 1,2,3. \]

Where,

\[ \rho_0 : \text{Fluid density.} \]

\[ \rho_1 : \text{Solid density (skeleton).} \]

\[ \rho_a : \text{Inertial coupling term defined by tortuosity (see equation (162)).} \]

Biot defines the kinetic energy of a porous material as the sum of three terms; that of the solid, of the fluid and a term corresponding to the coupling between both mediums.

\[ E_c = \frac{1}{2} \rho_{11} \left| \ddot{u}^f \right|^2 + \rho_{12} \dot{u}^f \ddot{u}^s + \frac{1}{2} \rho_{22} \left| \ddot{u}^s \right|^2. \quad (153) \]

Where,

\[ \rho_{11} = \rho_1 - \rho_{12}, \quad (154) \]

\[ \rho_{22} = \phi \rho_0 - \rho_{12}, \quad (155) \]

\[ \rho_a = -\rho_{12}. \quad (156) \]

If we apply Newton’s second law to a fluid of density, \( \rho_0 \), not viscous (see equation (78) with viscosity \( \eta = 0 \)) travelling in a circular section tube at a speed in the direction of the tube axis \( v_3(x_3) \), we obtain the following expression for inertial forces:

\[ \frac{\partial p}{\partial x_3} = -j \omega \rho_0 v_3(x_3). \quad (157) \]
Now, if we consider a porous medium with porosity of $\phi$ and cylindrical pores inclined at an angle of $\varphi$, as indicated in Figure 21, we obtain:

\[
\frac{\partial p}{\partial x} = \frac{1}{\cos \varphi} \frac{\partial p}{\partial x_3}, \quad (158)
\]

\[
\frac{\partial p}{\partial x} = -j \omega \phi \rho_0 v(x) \frac{1}{\cos^2 \varphi}. \quad (159)
\]

Using equation (46) for tortuosity in porous mediums with inclined cylindrical pores, we obtain:

\[
\frac{\partial p}{\partial x} = -j \omega \phi \rho_0 k_0 v(x). \quad (160)
\]

Where $k_0$ is the tortuosity. For general section pores and using the generalized nomenclature for tortuosity, this expression is equal to:

\[
\frac{\partial p}{\partial x} = -j \omega \phi \rho_0 \alpha v(x). \quad (161)
\]

On the other hand, it is seen that these forces of inertia will be equal to $\rho_{22} \ddot{u}^f$. Where $\ddot{u}^f = j \omega \ddot{u}^f = j \omega v(x)$, obtaining $\rho_{22} = \phi \alpha \rho_0$ and using equations (155) and (156) we obtain: the inertial coupling term ($\rho_a$) as a function of tortuosity,

\[
\rho_a = \rho_0 \phi (\alpha - 1). \quad (162)
\]
Substituting the volume forces for viscosity-dependent terms, the movement equations (151) and (152) can be expressed as follows:

$$(\rho_1 + \rho_a) \frac{\partial^3 u_i^s}{\partial t^2} - \rho_a \frac{\partial^2 u_i^s}{\partial t^2} = (P - N) \frac{\partial \theta^i}{\partial x_i} + \nabla \nabla^2 u_i^s + Q \frac{\partial \theta^i}{\partial x_i} - \sigma \phi^2 G(\omega) \frac{\partial}{\partial t} (u_i^s - u_i^f),$$

(163)

$$i = 1,2,3 .$$

$$(\phi \rho_a + \rho_a) \frac{\partial^3 u_i^f}{\partial t^2} - \rho_a \frac{\partial^2 u_i^f}{\partial t^2} = R \frac{\partial \theta^i}{\partial x_i} + Q \frac{\partial \theta^i}{\partial x_i} + \sigma \phi^2 G(\omega) \frac{\partial}{\partial t} (u_i^s - u_i^f),$$

(164)

$$i = 1,2,3 .$$

Where, $G(\omega)$ is $G_1(s)$ or $G_C(\omega)$ defined in equations (102) and (107).

Finally, considering the vectorial and harmonic shape of the displacements, $u = u e^{i\omega t}$, equations (163) and (164) are expressed:

$$-\omega^2 (\bar{p}_{11} u_i^s + \bar{p}_{12} u_i^f) = (P - N) \nabla \nabla . u^s + \nabla \nabla^2 u^s + Q \nabla \nabla . u^f ,$$

(165)

$$-\omega^2 (\bar{p}_{22} u_i^f + \bar{p}_{12} u_i^s) = R \nabla \nabla . u^f + Q \nabla \nabla . u^s .$$

(166)

Where,

$$\bar{p}_{11} = \rho_1 + \rho_a - j \sigma \phi^2 \frac{G(\omega)}{\omega},$$

(167)

$$\bar{p}_{12} = -\rho_a - j \sigma \phi^2 \frac{G(\omega)}{\omega},$$

(168)

$$\bar{p}_{22} = \phi \rho_a + \rho_a - j \sigma \phi^2 \frac{G(\omega)}{\omega}.$$  

(169)
In equations (163) and (164), the last term of the equation represents the force produced by fluid viscosity inside the porous medium. Substituting function $G_c(s)$ (see equation (107)) in equation (88), which is none other than Newton’s equation (78) with the solution under Bessel’s functions and taking into account that

$$s = \left( \frac{8\omega\rho_0}{\sigma\phi\cos^2\phi} \right)^{1/2}, \tag{170}$$

we obtain the following equation:

$$-\frac{\partial p}{\partial x_i} = j\omega\rho_3\nabla_3 - \sigma\phi G_c \nabla_3. \tag{171}$$

Comparing this equation with Newton’s equation (78), we obtain that the last term of equation (171) corresponds to the volume forces $(X_3)$. It should be noted that in equations (163) and (164) the $\phi^2$ factor appears instead of $\phi$, this is because in equation (171) they refer to forces per unit of areas, while in Biot’s equations they refer to volume forces, which is why $\phi^2$ appears.

**Waves in Biot’s model**

The displacement vector can be expressed as the sum of two vectors. The first addend can be obtained from a scalar potential (irrotational part), while the second can be obtained from a vectorial potential (rotational part):

$$u^s = u^{s,i} + u^{s,r}, \tag{172}$$

$$u^{s,i} = \nabla \phi^s, \tag{173}$$

$$u^{s,r} = \nabla \psi^s. \tag{174}$$

Fields $\phi^s$ and $\psi^s$ are the scalar and vectorial potentials associated with the displacement of the solid.

In like manner, the displacement of the fluid part can be divided into an irrotational and rotational part:
\( \mathbf{u}^f = \mathbf{u}^{\varphi} + \mathbf{u}^{\psi} \), \quad (175)

\( \mathbf{u}^{\varphi} = \text{grad} \varphi^f \), \quad (176)

\( \mathbf{u}^{\psi} = \text{rot} \psi^f \). \quad (177)

Taking into consideration the irrotational parts, that is, the scalar fields, starting with equations (165) and (166) and taking into account that \( \nabla \nabla^2 \varphi = \nabla^2 \nabla \varphi \), we can obtain the following equations:

\[-\omega^2 (\bar{\rho}_{11} \varphi^s + \bar{\rho}_{12} \varphi^f) = P \nabla^2 \varphi^s + Q \nabla^2 \varphi^f, \quad (178)\]

\[-\omega^2 (\bar{\rho}_{12} \varphi^s + \bar{\rho}_{22} \varphi^f) = Q \nabla^2 \varphi^s + R \nabla^2 \varphi^f. \quad (179)\]

If we write matrixes \( \rho \) and \( M \) as:

\[
\rho = \begin{bmatrix}
\bar{\rho}_{11} & \bar{\rho}_{12} \\
\bar{\rho}_{12} & \bar{\rho}_{22}
\end{bmatrix},
\]

\[
M = \begin{bmatrix}
P & Q \\
Q & R
\end{bmatrix}.
\]

Equations (178) and (179) can be expressed as follows:

\[-\omega^2 [M]^{-1} \rho \nabla \varphi = \nabla^2 \varphi. \quad (182)\]

If we calculate the eigenvalues \( \delta_1^2 \) and \( \delta_2^2 \) of the left side matrix of this equation, equation (182) is reduced to two Helmholtz equations that must be satisfied by eigenvectors \( \varphi_1 \) and \( \varphi_2 \):

\[-\delta_1^2 \varphi_1 = \nabla^2 \varphi_1, \quad (183)\]

\[-\delta_2^2 \varphi_2 = \nabla^2 \varphi_2. \quad (184)\]

This gives us the pressure waves equation:
\[ \nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}. \] (185)

Where, pressure is written as \( p = \tilde{p}e^{i\omega t} \) and deriving this expression twice in time, we obtain the following expression for equation (185):

\[ \nabla^2 p = -\frac{\omega^2}{c^2} \tilde{p} = -k^2 \tilde{p}. \] (186)

Comparing equation (186) with equations (183) and (184) we conclude that the eigenvalues \((\delta_1^2 \text{ and } \delta_2^2)\) are the numbers of waves of the compression waves. Therefore, we obtain the complex velocities of each pair of waves as,

\[ c_{i,1} = \sqrt{\frac{2(PR - Q^2)}{P\bar{\rho}_{22} + R\bar{\rho}_{11} - 2Q\bar{\rho}_{12} - \sqrt{\Delta}}} , \] (187)

\[ c_{i,2} = \sqrt{\frac{2(PR - Q^2)}{P\bar{\rho}_{22} + R\bar{\rho}_{11} - 2Q\bar{\rho}_{12} + \sqrt{\Delta}}} . \] (188)

Where,

\[ \Delta = (P\bar{\rho}_{22} + R\bar{\rho}_{11} - 2Q\bar{\rho}_{12})^2 - 4(PR - Q^2)(\bar{\rho}_{11}\bar{\rho}_{22} - \bar{\rho}_{12}^2) . \] (189)

As a result, these equations prove that there are two types of waves that propagate at different speeds:

1) One compression or expansion wave \( \mathbf{u}^{(i)}_1 \) in the fluid (respectively \( \mathbf{u}^{(i)}_1 \) in the solid) that is propagated at a complex speed of \( c_{i,1} \).

2) One compression or expansion wave \( \mathbf{u}^{(i)}_2 \) in the fluid (respectively \( \mathbf{u}^{(i)}_2 \) in the solid) that is propagated at complex speed \( c_{i,2} \).
The characteristic impedance in the direction $x_3$ (see Figure 18) taking into consideration that the macroscopic movement occurs in this direction $x_3$, 

$$Z_i = \frac{p}{j\omega u_3}, \quad (190)$$

From equation (178) we obtain:

$$\mu_i = \frac{\phi_i}{\varphi_i} = \frac{P\delta_i^2 - \omega^2 \rho_i}{\omega^2 \rho_{12} - Q\delta_i^2} \quad i = 1,2, \quad (191)$$

that represents a ratio of skeleton and fluid speeds.

By using equations (121) and (147), and taking into account that the displacement of a compression wave is $u_j = u e^{j k x} e^{j\omega t}$, and having determined that $\delta = k$; equation (190) is rewritten as:

$$Z_{1i} = \left(R + \frac{Q}{\mu_1}\right)\frac{\delta_i}{\phi \omega}, \quad (192)$$

$$Z_{2i} = \left(R + \frac{Q}{\mu_2}\right)\frac{\delta_2}{\phi \omega}. \quad (193)$$

However, the characteristic impedance associated with sound propagation in the skeleton can be expressed as seen in equation (194):

$$Z^s = -\frac{\sigma_{33}^s}{j\omega u_3}. \quad (194)$$

In an analogous manner, by using equations (120) and (147) and taking into consideration that displacement in a compression wave is $u_j = u e^{j k x} e^{j\omega t}$, equation (194) can be rewritten for the two compression waves with different complex velocities:

$$Z_{1i} = \left(P + Q\mu_1\right)\frac{\delta_i}{\omega}, \quad (195)$$
\[ Z_2' = (P + Q\mu_2)\frac{\delta_{\nu}}{\omega}. \] (196)

The same way we have performed the operation until now to obtain compressions waves, by using the irrotational part of the movement equations, if we only consider the rotational part of equations (165) and (166):

\[-\omega^2(\bar{\rho}_{11}\psi + \bar{\rho}_{12}\psi^f) = N\nabla^2\psi', \] (197)

\[-\omega^2(\bar{\rho}_{12}\psi^f + \bar{\rho}_{22}\psi^f) = 0.\] (198)

If we solve the vectorial potential of the fluid, \(\psi'\), from equation (198), we can write equation (197) as a Helmholtz equation,

\[ \nabla^2\psi' + \frac{\omega^2}{N}\left(\frac{\bar{\rho}_{11}\bar{\rho}_{22} - \bar{\rho}_{12}^2}{\bar{\rho}_{22}}\right)\psi' = 0. \] (199)

Where the complex propagation velocity of the wave is:

\[ c_r = \sqrt{\frac{N\bar{\rho}_{22}}{\bar{\rho}_{11}\bar{\rho}_{22} - \bar{\rho}_{12}^2}}. \] (200)

Therefore, equations (197) and (198) prove the existence of a wave in each medium that propagates at the same complex velocity \(c_r\): A shear wave in the fluid, \(u^r\), and the corresponding shear wave in the solid, \(u^{sr}\).

4. COMPARISON BETWEEN STANDING WAVE TUBE MEASUREMENTS AND THEORETICAL CALCULATIONS

In this section we are going to report results of the measurements of the surface impedance in a tube of impedance made in Ikerlan according to ISO 10534-2: 1998, the surface impedance measured in the CTAG (Centro Tecnológico de Automoción de Galicia) laboratories using a commercial standing
wave tube (Bruël and Kjaer Type 4206) and the impedance measured in laboratories of Katholieke Universiteit Leuven (Belgium).

Furthermore, in the laboratories of Katholieke Universiteit Leuven (Belgium) all parameters defined in section 2 have been measured. Hence, we have implemented the Allard-Biot model in Matlab in order to calculate the surface impedance of an absorbing material and to compare these theoretical results with the measurements mentioned above.

The measurements and the calculation have been performed in three absorbing materials: Acustec, Acustifiber P and Acusticell. Acustec is a material made of mineral wool with a high mechanical strength, meanwhile Acustifiber P is made of polyester fibre and Acusticell is an absorbing expanded polyurethane foam. The data given by the supplier show the following thicknesses and densities for the different materials:

- 25 mm of thickness and 2.2 kg m-2 of surface density for Acustec.
- 40 mm of thickness and 15 kg m-3 of density for Acustifiber P.
- 25 mm of thickness and 30 kg m-3 of density for Acusticell.

The value of the poroelastic parameters of these materials is shown in table 3.

<table>
<thead>
<tr>
<th>Porous</th>
<th>Acustec</th>
<th>Acustifiber P</th>
<th>Acusticell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (10⁻³ m)</td>
<td>24.4</td>
<td>39</td>
<td>24</td>
</tr>
<tr>
<td>Air flow resistivity (Ns/m²)</td>
<td>29000</td>
<td>2000</td>
<td>22000</td>
</tr>
<tr>
<td>Porosity ()</td>
<td>&gt; 0.95</td>
<td>&gt; 0.95</td>
<td>&gt; 0.95</td>
</tr>
<tr>
<td>Tortuosity ()</td>
<td>1.08</td>
<td>1.03</td>
<td>1.38</td>
</tr>
<tr>
<td>Viscous length (10⁻⁶ m)</td>
<td>30</td>
<td>420</td>
<td>17</td>
</tr>
<tr>
<td>Parameter</td>
<td>Lab 1</td>
<td>Lab 2</td>
<td>Lab 3</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>Thermal length ($10^{-6}$ m)</td>
<td>80</td>
<td>650</td>
<td>40</td>
</tr>
<tr>
<td>Skeleton density (kg/m³)</td>
<td>69</td>
<td>17</td>
<td>26</td>
</tr>
<tr>
<td>Young’s modulus (kPa)</td>
<td>1060</td>
<td>13</td>
<td>192</td>
</tr>
<tr>
<td>Structural loss factor</td>
<td>0.08</td>
<td>0.23</td>
<td>0.13</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.0</td>
<td>0.0</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 3: The values of the parameters of the absorbing materials measured in KUL laboratories.

The material named Acustec has an impervious screen. This screen can be considered as a very thin porous material (0.0001 m) with a very high flow resistivity ($2 \times 10^7$ Ns/m⁴). The effect of the screen on the absorption coefficient is the appearance of a resonance peak related to the elasticity of the porous material. The clamping of the sample in the tube, adds to the rigidity and the corresponding damping of the sample. Further information can be found in Vigran et al (1997). The screen was very carefully removed. The material parameters of Acustec that are provided in Table 3 were determined without the screen.

The comparison between measured normal absorption coefficient in the three laboratories and the calculated one for the three poroelastic materials is herewith shown. For the material Acustec, only the second laboratory (lab2) measured with and without the impervious material. It is important to underline the third laboratory (lab3) carried out the characterization of the material until 1400 Hz.
Moreover, the comparison between the real and imaginary part of the measured normalized acoustic surface impedance and calculated one is done.
Figure 27. Comparison of real part of the normalized surface impedance for the mineral wool Acustec with impervious screen.

Figure 28. Comparison of imaginary part of the normalized surface impedance for the mineral wool Acustec with impervious screen.

Figure 29. Comparison of real part of the normalized surface impedance for the mineral wool Acustec without impervious screen.

Figure 30. Comparison of imaginary part of the normalized surface impedance for the mineral wool Acustec without impervious screen.
The results suggest that there are significant differences between the inter-laboratory measurements and also between the measurements and the calculations. Those differences, of course, depend on the poroelastic material. So, there is no noticeable dispersion in the normalized acoustic surface impedance in the case of the polyester fibre from 300 Hz until 2400 Hz. The most important difference is in the value of the resonant frequency of the material: around 3700 Hz for lab2, 4700 Hz for lab1 and around 4100 for the calculation. Same results regarding the value of resonant frequency we have in the case of expanded polyurethane foam. But, now, we have substantial dispersion in the real part of the normalized surface impedance until 1600 Hz. The case of the mineral
wool is completely different. Since Acustec has an impervious screen, the Kundt tube is not a very accurate way to measure its acoustic surface impedance. In Figure 27 we can see the large dispersion for the value of the real part of normalized surface impedance. However, the agreement between the imaginary data is better, but in any case the dispersion is noticeable. Until the resonant frequency value, the measured imaginary part of the surface impedance of Acustec in the three laboratories is quite similar. The resonant frequency effect is better characterized by lab2, although after that frequency lab1 shows better agreement with the calculated surface impedance. According to Figure 23, the measurements of the normal absorption coefficient in the second laboratory are closer to the calculated one, at least in its general tendency. Attending to the mineral wool without impervious screen, again we can see large dispersion in the real part and a quite good agreement in the imaginary part. It is important to underline the dispersion in the real part is not easy to understand since the tortuosity and resistivity values for the mineral wool without screen are not high enough. We think being the mineral wool stiff enough, the boundary effect in the interior of the wave standing wave could influence the value of its normalized acoustic surface impedance.

Other interesting comparison is the one between different poroelastic material characterization models. We use three models. The first one is the Allard-Biot generalized model which considers the elasticity of the frame (used before when we compared normalized surface impedance calculations versus inter-laboratory data and explained in section 3.4.1), the second one is the Allard-Champoux model (explained in section 3.1.3) and the third one is the first formulation of the Allard model which considers the frame as stiff (explained in section 3.3.1).
Figure 35. Comparison of real part of the normalized surface impedance for the mineral wool Acustec with impervious screen.

Figure 36. Comparison of imaginary part of the normalized surface impedance for the mineral wool Acustec with impervious screen.

Figure 37. Comparison of real part of the normalized surface impedance for the expanded polyurethane foam Acusticell.

Figure 38. Comparison of imaginary part of the normalized surface impedance for the expanded polyurethane foam Acusticell.
Analysing the results of prediction, we can conclude that all three models give us similar results in the case of the polyester fibre, although the value of the measured (in lab2) resonant frequency and the predicted one do not agree. In the case of the polyurethane foam Acusticell we can confirm Allard-Champoux model is not valid since the elasticity of the frame needs to be considered. The results for this case suggest that the measured surface impedance in lab2 shows better agreement with the Allard First formulation model than Allard-Biot model. In contrast, the three models provide quite different results for the mineral wool Acustec with its impervious screen. The models not considering the elasticity of the frame give us similar results but they are very different to the results offered by the generalized Allard-Biot model. The measured results by means of the standing wave method for this kind of material are not accurate and according to Lauriks et al (1990) (where the theoretical results are validated versus free-field techniques) we can conclude that to describe the mineral wool Acustec with its impervious screen we need a model considering the elasticity of the frame.

5. CONCLUSIÓN

In Section 2 we describe the poroelastic materials using the parameters that define each of the two phases or mediums that comprise poroelastic material; the fluid phase, that is normally air, and the solid phase, or skeleton. For correct characterization of these materials, besides knowing the various
parameters that define the two phases, it is necessary to know the parameters that define the coupling between the two phases.

If the parameters that define the properties of the fluid are density, the relation of specific heats, the speed of sound, dynamic viscosity and Prandtl's number; the parameters that define the solid or skeleton are basically Young's complex model and Poisson's coefficient.

It is important to bear in mind the parameters that define coupling between the two phases of poroelastic material. These parameters are porosity, resistivity, tortuosity and viscous and thermal characteristic lengths. We know that at high frequencies, parameters of porosity, tortuosity and characteristic lengths play a very important role, while at low frequencies, porosity, air flow resistivity and thermal permeability defined by Lafarge et al (1997) are the key parameters. Therefore, we can see that porosity is the parameter that plays an important role in the complete range of frequencies.

All these parameters appear in Biot's general model.

In Section 3 we present the most representative models that characterize poroelastic materials. The characterization of sound propagation in a poroelastic material can be performed on a microscopic scale, that is, if the pores are considered cylindrical on a scale of the distance between pore axes. However, due to the complexity of the geometry of poroelastic material skeletons, study at a microscopic scale is very difficult to perform. Therefore, work is done using the quantities that are related with sound propagation. This is carried out on a macroscopic scale on volumes that present dimensions that are large enough for the measurements to be significant. At the same time, however, these dimensions must be much smaller than the wavelength.

Even on a macroscopic level, study of sound propagation in poroelastic material is very difficult to perform. For this purpose, there are empirical models that characterize porous materials, such as that of Delany and Bazley (1970) (later corrected by Mechel-Ver (Chapter 8 of Beranek and Ver (1992)) and Allard and Champoux (1992)). Delaney and Bazley (1970) wrote laws for fibrous absorbing materials that described impedance and the constant of propagation characteristic of material as a function of the quotient between frequency and resistivity. Mechel-Ver (Chapter 8 of Beranek and Ver (1992)) rewrote these same laws as a function of an adimensional parameter (density by the quotient between frequency and resistivity) and distinguished two type of material and
the frequency range. Lastly, Allard and Champux (1992) improve the range of the lowest frequencies, taking into account the thermal effect dependent on frequency.

Zwikker and Kosten (1949) developed a model that considered the pores as having a circular section. The study of the viscous and thermal effects, the latter studied by Stinson (1991), determines the dynamic density and compressibility modulus necessary for the definition of characteristic impedance and propagation constant of the material.

On the other hand, there are phenomenological models of equivalent fluid that assume that the skeleton is rigid and poroelastic phenomenological models that assume that the skeleton is elastic. All the coupling parameters intervene in the equivalent fluid models (porosity, tortuosity, air flow resistivity, thermal and viscous characteristic lengths). The model that best describes skeleton elasticity is Biot’s model (1956), which also makes use of the parameters mentioned previously as well as of the parameters of the skeleton itself (Young’s modulus, Poisson’s modulus, shear modulus and damping).

Biot’s general model (described in subsection 3.4.1) defines movement equations for displacement and deformation tensors of the fluid and the solid, or skeleton. These equations include the parameters that describe coupling between the fluid and the skeleton. These coefficients can be identified with physical properties such as the fluid compressibility modulus, the elastic skeleton compressibility modulus, porosity or the shear modulus. Biot also introduces coupled density as a function of tortuosity. Once these coefficients are determined, the movement equations can be resolved and both impedance and propagation velocity can be determined for each of the waves in each phase or medium of the material.

Starting from the equations of skeleton and fluid movement, Biot defines three waves: 2 compression waves and one shear wave.

Allard (1993) states that:

“For the case where a strong coupling exists between the fluid and the frame, the two compressional waves exhibit very different properties, and are identified as the slow wave and the fast wave (Biot (1956b) and Johnson et al. (1987)). The ratio $\mu$ of the velocities of the fluid and the frame is close to 1 for the fast wave, while these velocities are nearly opposite for the slow wave. The damping due to viscosity is much stronger for the slow wave
which, in addition, propagates more slowly than the fast wave. With ordinary porous materials saturated by air, the coupling between air and frame is not great enough for these properties of the compressional waves to be verified. It is more convenient to refer to them as a frame-borne wave and airborne wave. This new nomenclature is obviously fully justified if there is no coupling between frame and air. For such a case, one wave propagates in the air and the other in the frame. For the case where a weak coupling exists, the partial decoupling previously presented by Zwiker and Kosten (1949) occurs. With the frame being heavier than air, the frame vibrations will induce vibrations of the air in the porous material, yet the frame can be almost motionless when the air circulates around it. More precisely, one of the two waves, the airborne wave propagates mostly in the air, whilst the frame-borne wave propagates in both media. If the bulk modulus of the frame, $K_b$, is also larger than the bulk modulus of the air, the wave number of the frame-borne wave and its characteristic impedance corresponding to the propagation in the frame can be close to the wave number and the characteristic impedance of the compressional wave in the frame when in vacuum. The shear wave is also a frame-borne wave, and is very similar to the shear propagating in the frame when in vacuum (page 133)."

Characteristic impedance of fibrous material is shown in Figure 41. The material is a layer of glass wool.

Figure 41: Impedance of a layer of fibrous material measuring 10 mm thick. Prediction with Biot's theory___ Prediction for the same material but with a rigid skeleton ----. Measurements ••• Allard (1993)
The rigid skeleton model is valid in general terms, but there are certain deviations between Biot’s prediction and the prediction for the same material with a rigid skeleton, as shown in Figure 41 at around 500 Hz. These deviations are due to the fact that rigid skeleton models, by definition, do not include skeleton resonances, but are included in Biot’s model. Therefore, Biot’s theory offers better results for these resonance frequencies.

In the section 4 of this article we have proved these conclusions where three poroelastic materials have been described with three different models.

Finally, it is important to underline that general models exist that are based on homogeneization methods that associate microscopic and macroscopic sound propagation. These more complex mathematical models have not been covered in this review. In addition, according to Allard (1983):

“For the porous materials having a flow resistivity smaller than 50000 Nm-4s, the two theories are compatible at least at frequencies higher than 100 Hz [section 1 of Burridge and Keller (1985)] (page 118).”

1 The mechanical impedance of a simple resonator when a force, $f$, is applied whose harmonic frequency is $\omega$ is defined as $z = f e^{i\omega t} / x e^{i\omega t} = (s - \omega^2 m) + j r \omega$, where the mass is $m$, $r$ is the damping and $s$ is the stiffness. Being the mechanical reactance the imaginary part of the mechanical impedance, for the simple resonator case under forced excitation it results $r \omega$.

2 The neper unit is similar to bel, but use logarithm to base e instead of logarithm to base 10.

3 There is an error in the formulation of the compressibility modulus by Attenborough (1987) written by Allard (1993), as $J(\sqrt{s - j})$ should be $J(Bs \sqrt{-j})$.

4 There is an error in the formulation of the skeleton shear equation given by Biot, taken from Allard (1993), as $\tau_{ij}^s$ should be $\sigma_{ij}^s$.

5 Recalling the following algebraic ratios:

$$\nabla \cdot \nabla \alpha = \nabla^2 \alpha = \text{div } \text{grad } \alpha \quad \nabla \cdot \mathbf{u} = \text{grad } \text{div } \mathbf{u}$$

where,

$$\nabla \alpha = \text{grad } \alpha$$

$$\nabla \cdot \mathbf{u} = \text{div } \mathbf{u}$$

6 There is an error in the formulation by Allard (1993) in the equation of compression waves considering only the rotational part, as $-\omega^2 \bar{p}_{12} \psi^s - \omega \bar{p}_{22} \psi^f = 0$ should be $-\omega^2 \bar{p}_{12} \psi^s - \omega^2 \bar{p}_{22} \psi^f = 0$. 

Bibliography


